# Theory and Practice of Free-Electron Lasers

## Particle Accelerator School
## Day 2

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# Course Content

# Chapter 4
# Optical Architectures

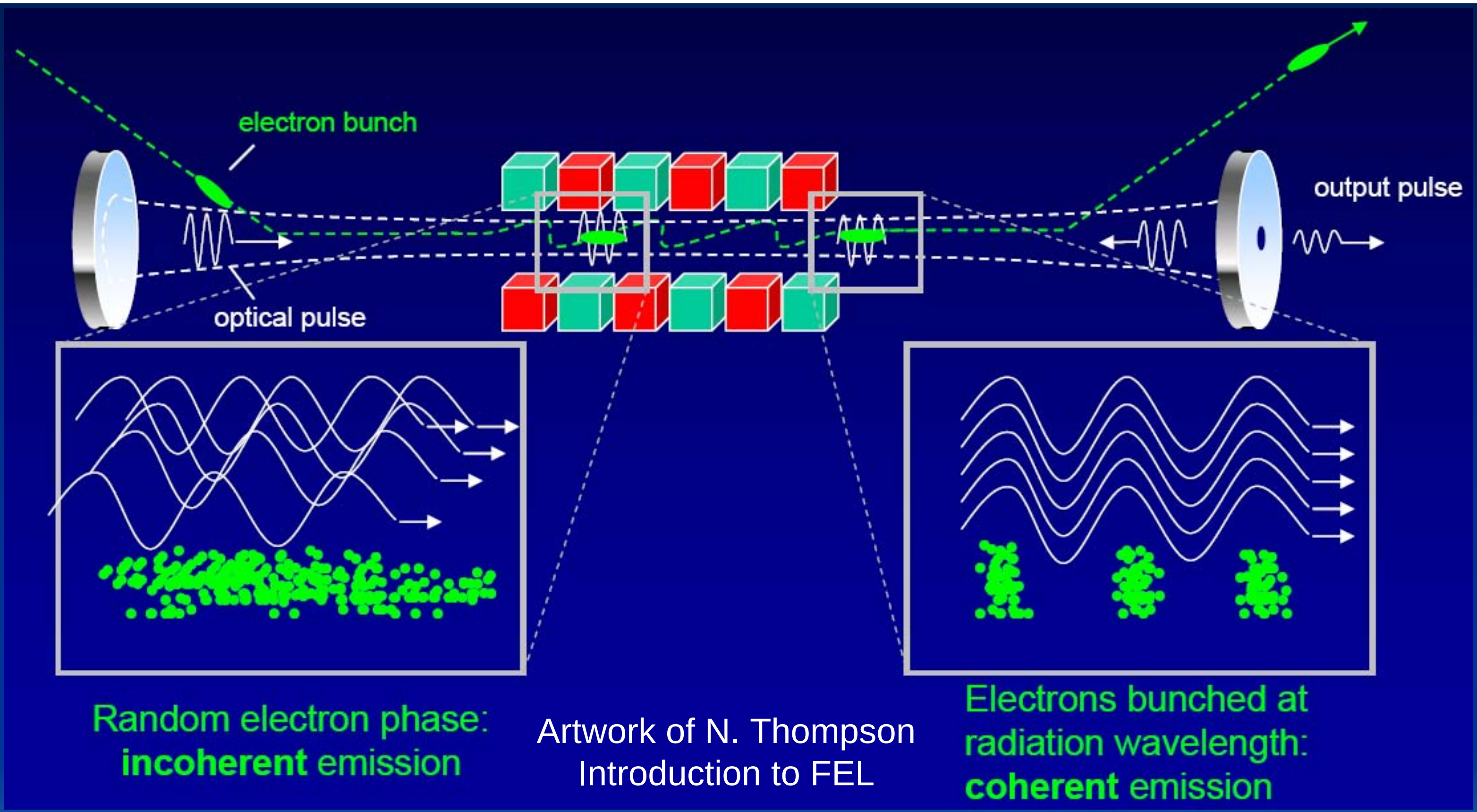
# Optical Architectures

- Oscillator FEL
- Regenerative Amplifier
- Self-Amplified Spontaneous Emission
- Seeded Amplifier
- Optical Klystron
- High-gain Harmonic Generation

# Oscillator FEL

Oscillator FEL uses a combination of low-current electron beam and an optical cavity to trap the optical pulse in several passes through the wiggler. Each pass involves a new electron bunch interacting with the optical field and undergoing microbunching at the end of the wiggler. The optical intensity grows by a small amount (low gain) in each pass with an amplification factor of 1 + $g_{ss}$. The **small-signal gain**, $g_{ss}$, is proportional to the cube of $N_w$ (number of wiggler periods). At high intensity, the electrons rotate in phase space and absorb light near the end of the wiggler, and the large-signal gain is reduced compared to $g_{ss}$. Saturation occurs when gain is equal to total cavity losses. Beyond saturation, the electrons' synchrotron oscillations lead to **sideband generation**. Oscillator FEL's extraction efficiency is approximately one divided by 2.4$N_w$. The most common optical cavity is the near-concentric resonator. At low gain, the optical mode is determined almost entirely by the resonator optics. The oscillator FEL cavity length has to be accurate to within a few optical wavelengths.

# Basic Configuration of an Oscillator FEL




FEL power builds up from noise to saturation in N passes inside an optical cavity. In the $N^{th}$ pass, a fresh bunch of randomly distributed electrons interacts with the optical beam and develops microbunching with period = $\lambda$.

# Small-Signal Gain

$$g_{ss} = \frac{2\pi N_w^3}{\gamma^3}\left(\frac{[JJ]a_w\lambda_w}{r_b}\right)^2\left(\frac{I}{I_A}\right)$$

| | |
|---|---|
| $N_w$ | number of wiggler periods |
| $\gamma$ | electron beam's Lorentz factor (~ 2 x energy in MeV) |
| *[JJ]* | reduction factor of FEL interaction in a planar wiggler |
| $a_w$ | dimensionless wiggler parameter |
| $\lambda_w$ | wiggler period |
| $r_b$ | electron beam radius in the wiggler |
| $I$ | electron beam peak current |
| $I_A$ | Alfven current; 17 kA |
| Note: | Amplification factor ($P_{out}/P_{in}$) = 1 + $g_{ss}$ |

**Small-signal gain must be higher than total round-trip losses.**

# Outcoupling

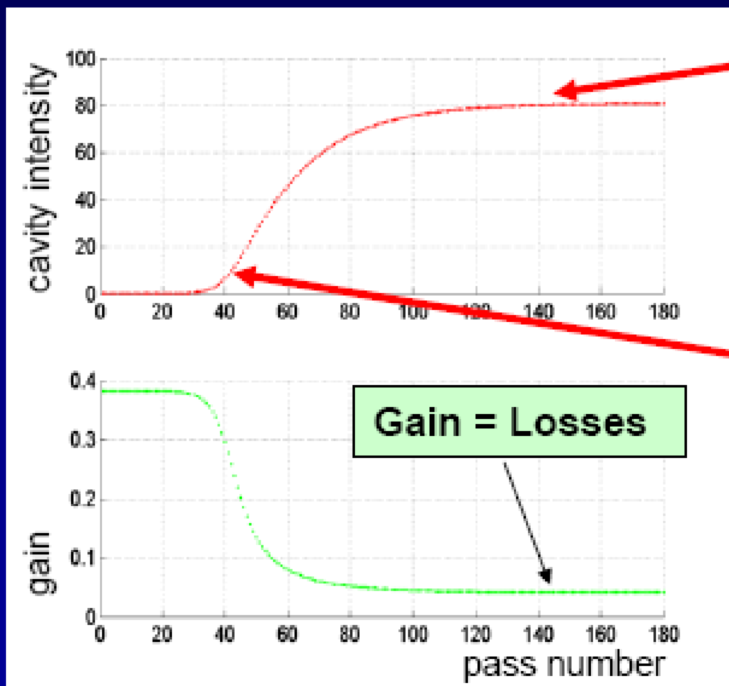

$$P_{out} = \left(1 - R_{outcoupler}\right)\left(1 - \Gamma_{absorption}\right) P_{cavity}$$

$$\boxed{P_{out} = \Gamma_{out} P_{cavity}}$$

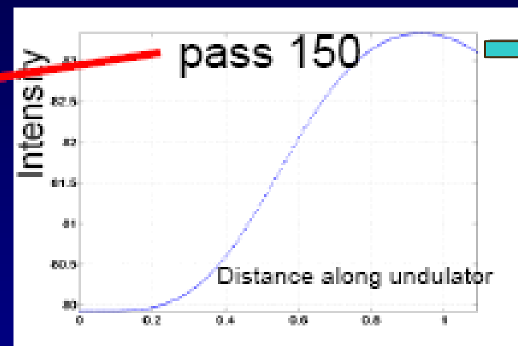
- Out-coupler's reflectivity is typically 80-99% (outcoupling ~ 1-20%).
- For maximum output power, absorption loss must be kept very low.
- The **intracavity power** is $(1 - R_{outcoupler})^{-1}$ times the output power.

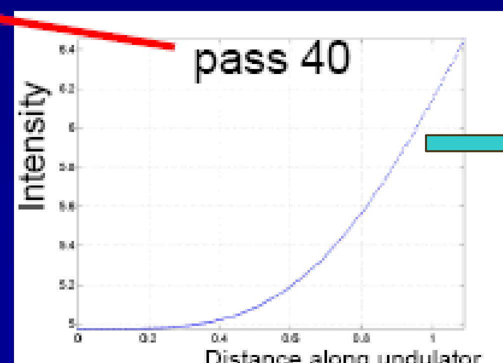
# Power Growth Curves

$g_{ss}$ = 39%   total round-trip losses = 4%




Power and gain vs pass number




Power vs z

At pass #150 near the wiggler exit, the electrons over-bunch and absorb light as they move up the bucket. Large-signal gain is smaller than small-signal gain.

At pass #40, intracavity power increases with $z^3$.

$P_{out} = P_{in}\,(1 + g_{ss})$

Courtesy of N. Thompson Introduction to FEL

As the optical intensity increases, the large-signal gain decreases until FEL gain is equal to the total cavity losses and FEL power saturates.

# Small-Signal Gain (revisited)

The small-signal gain for a planar wiggler at the peak of the gain curve, assuming the electron beam radius $\sigma$ is smaller than the optical beam, is

$$\boxed{g_{ss} = \left(\frac{2\pi N_w}{\gamma}\right)^3 \left(\frac{[JJ]a_w}{\sigma k_w}\right)^2 \left(\frac{I}{I_A}\right)}$$

Recall the dimensionless FEL (Pierce) parameter

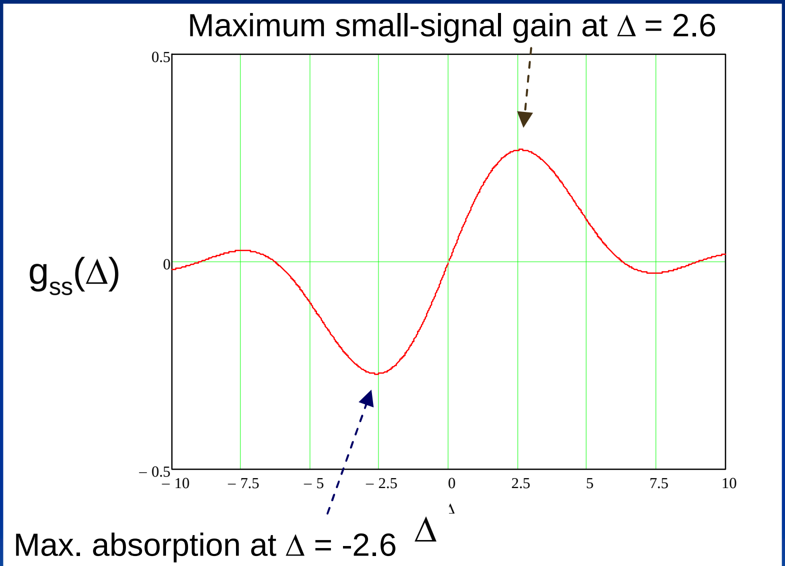
$$\rho = \frac{1}{2\gamma}\left(\frac{[JJ]a_w}{\sigma k_w}\right)^{\frac{2}{3}} \left(\frac{I}{I_A}\right)^{\frac{1}{3}}$$

The small-signal gain is proportional to the cube of the Pierce parameter

$$\boxed{g_{ss} = 2\left(2\pi N_w \rho\right)^3}$$

# Maximum Gain & Wiggler Efficiency

For maximum gain, one must select an energy detuning (electron beam energy minus FEL resonance energy) to be slightly positive. Conversely, at a fixed beam energy, the lasing wavelength is longer than the resonance wavelength.




$$\Delta = 4\pi N_w \left(\frac{\Delta E}{E}\right) = 2\pi N_w \left(\frac{\Delta\lambda}{\lambda}\right)$$

$$\Delta_{max} = 2.6$$

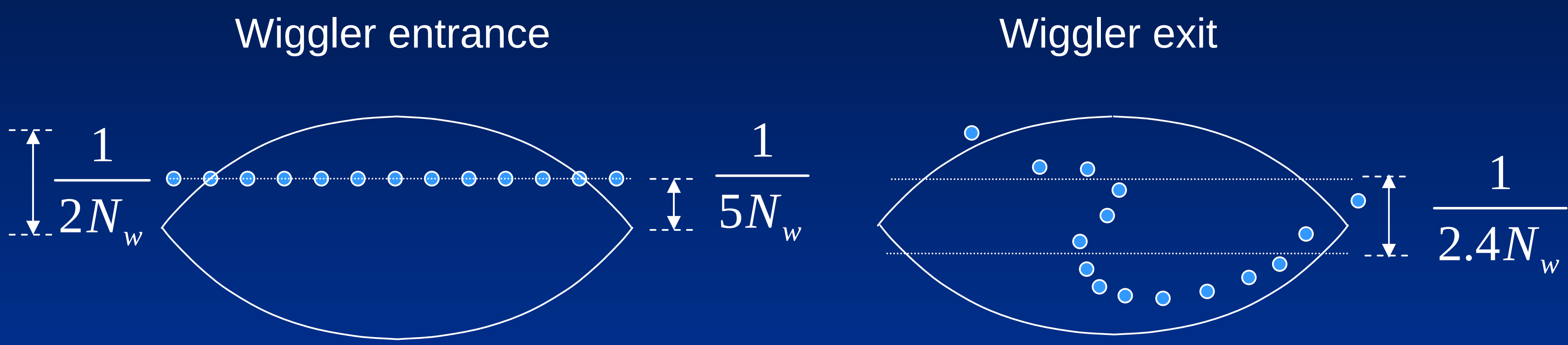
$$\left(\frac{\Delta E}{E}\right)_{max} = 2\Delta_{max} = \frac{2(2.6)}{4\pi N_w}$$

Wiggler efficiency of oscillator FEL

$$\boxed{\eta = \frac{P_{FEL}}{P_{e-beam}} = \frac{1}{2.4 N_w}}$$

Saturation occurs when average electron energy evolves from Δ = 2.6 (maximum gain) to Δ = -2.6 (maximum absorption).

# Visualization of Wiggler Efficiency




Oscillator FEL picks the wavelength for maximum gain ($\Delta\lambda/\lambda_R = 2.4/N_w$)

Wiggler extraction efficiency at saturation

$$\boxed{\eta = \frac{1}{2.4N_w}}$$

<u>Note</u>: A common mistake is to calculate the intra-cavity power from the wiggler extraction efficiency and electron beam power. In reality, the intra-cavity power is Q times the output power, where Q is the optical cavity quality factor (the inverse of 1 – reflectivity).

# Maximum Output Efficiency & Power

Output efficiency depends on wiggler efficiency, out-coupling and total losses

$$\eta_{FEL} = \left(\frac{\Gamma_{out}}{\Gamma_{out} + \Gamma_{loss}}\right)\frac{1}{2.4N_w}$$

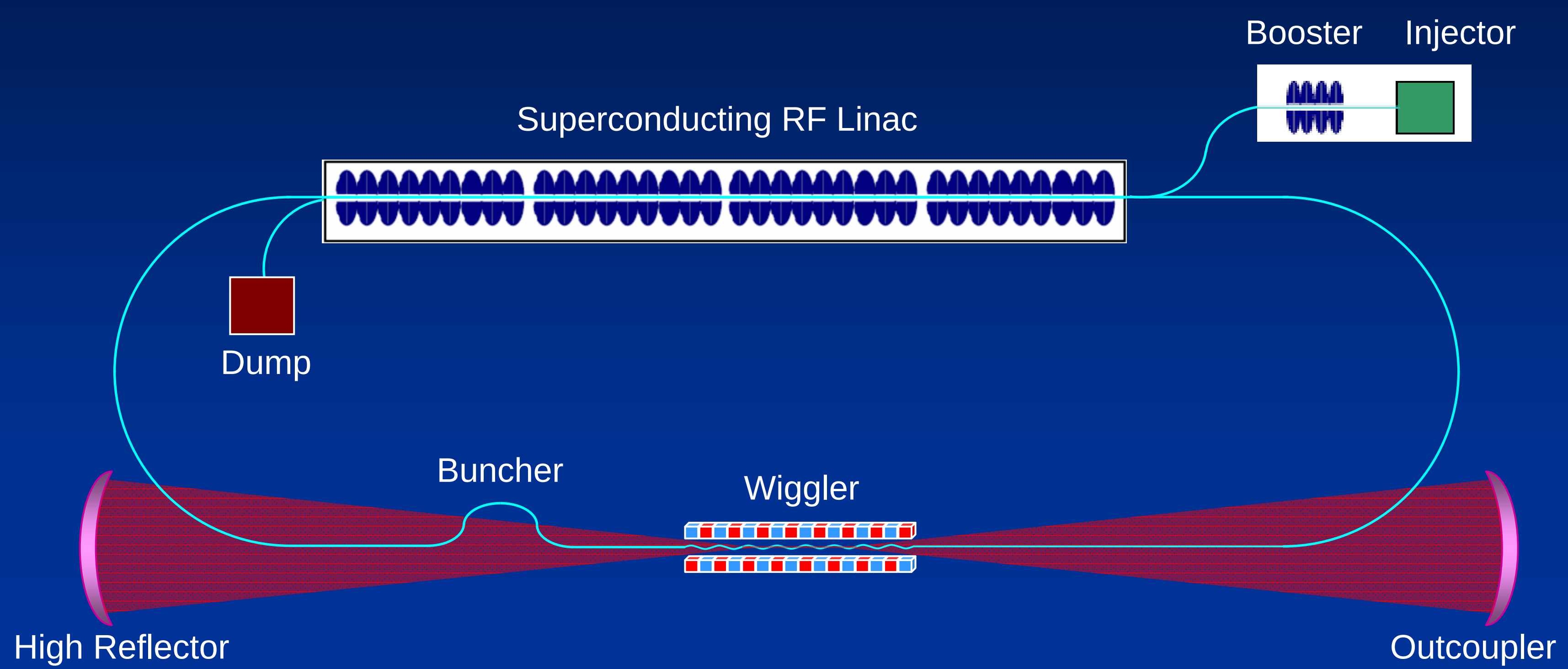
Maximum output power is electron beam power times output efficiency

$$P_{\max} = \left(\frac{\Gamma_{out}}{\Gamma_{out} + \Gamma_{loss}}\right)\frac{P_{beam}}{2.4N_w}$$

Maximum intracavity power

$$P_{cavity} = \left(\frac{1}{\Gamma_{out} + \Gamma_{loss}}\right)\frac{P_{beam}}{2.4N_w} = Q_{cavity}\frac{P_{beam}}{2.4N_w}$$

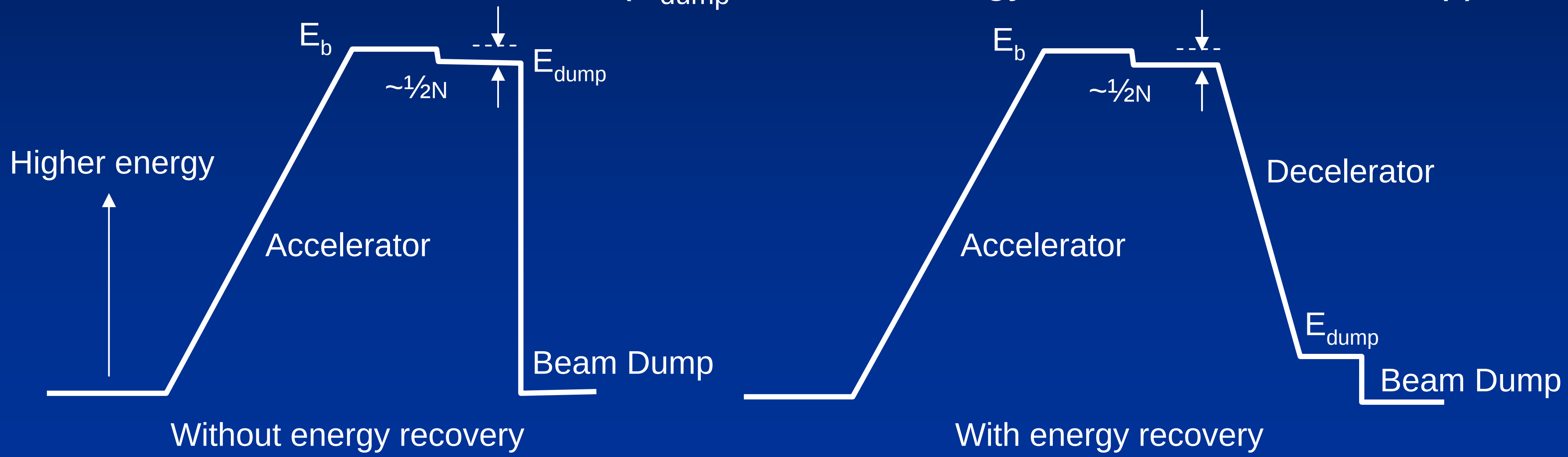
# Oscillator with Energy Recovery Example: Jefferson Lab FEL




Oscillator FEL's wall-plug efficiency can be significantly increased with energy recovery. The spent electron beams are circulated through the superconducting RF linac at the deceleration phase and dumped at lower energy to improve overall efficiency and reduce radiation hazards.

# Energy Recovery

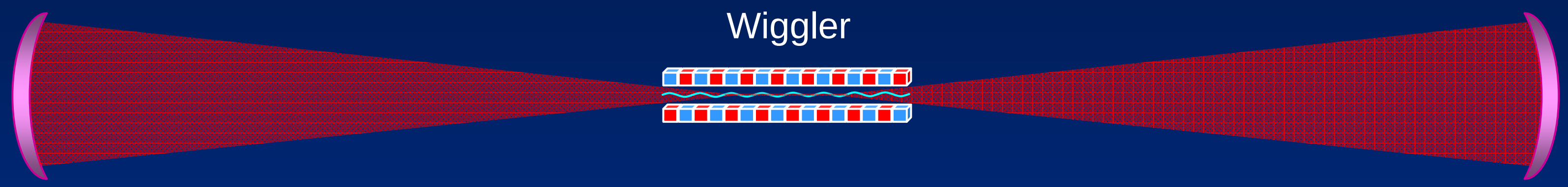
Oscillator FEL efficiency is typically 1%. The spent electron beams still have ~99% of the initial energy. Dumping the high-power electron beam is wasteful and creates radiation hazards ($E_{dump}$ is beam energy before the beam dump).



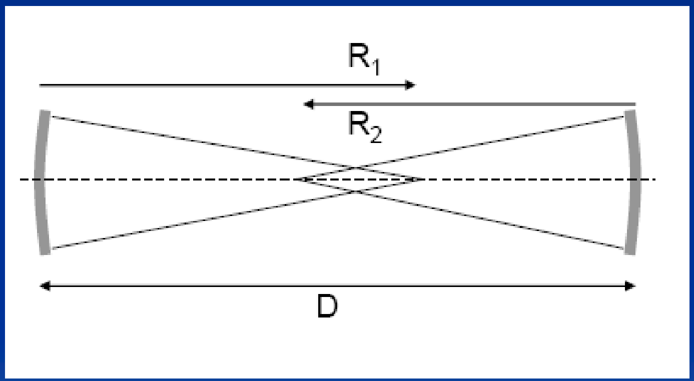

With energy recovery, the efficiency of electron-to-FEL conversion is enhanced by the ratio of beam energy at the wiggler to beam dump energy.

$$\boxed{\eta_{electron-FEL} = \frac{E_b}{2.4 N_w E_{dump}}}$$

# Optical Resonator

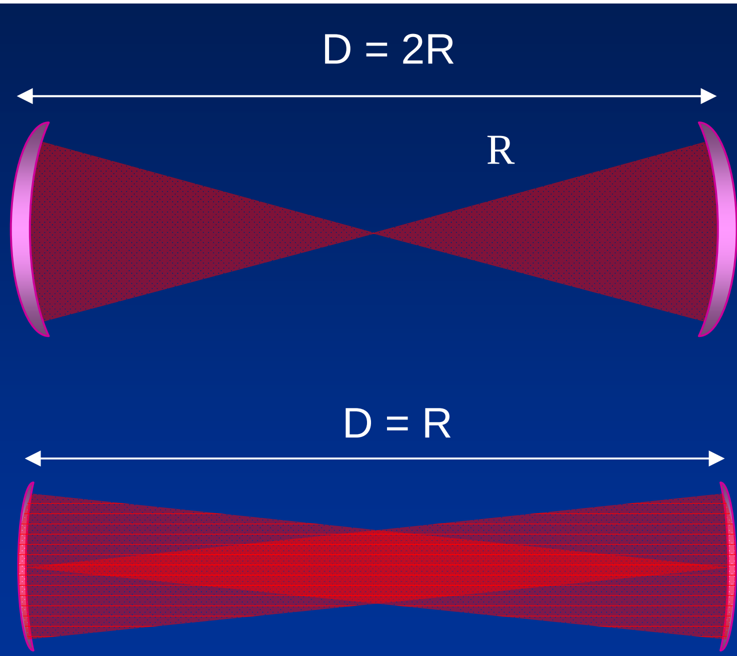



- The optical resonator consists of two concave mirrors with radii of curvature $R_1$ and $R_2$. The mirrors are separated by a distance D.




- A measure of stability is the overlap between the two radii of curvature.
- The Rayleigh length is a measure of the FEL beam's divergence. The shorter the Rayleigh length, the larger the divergence.

# Optical Resonator Designs



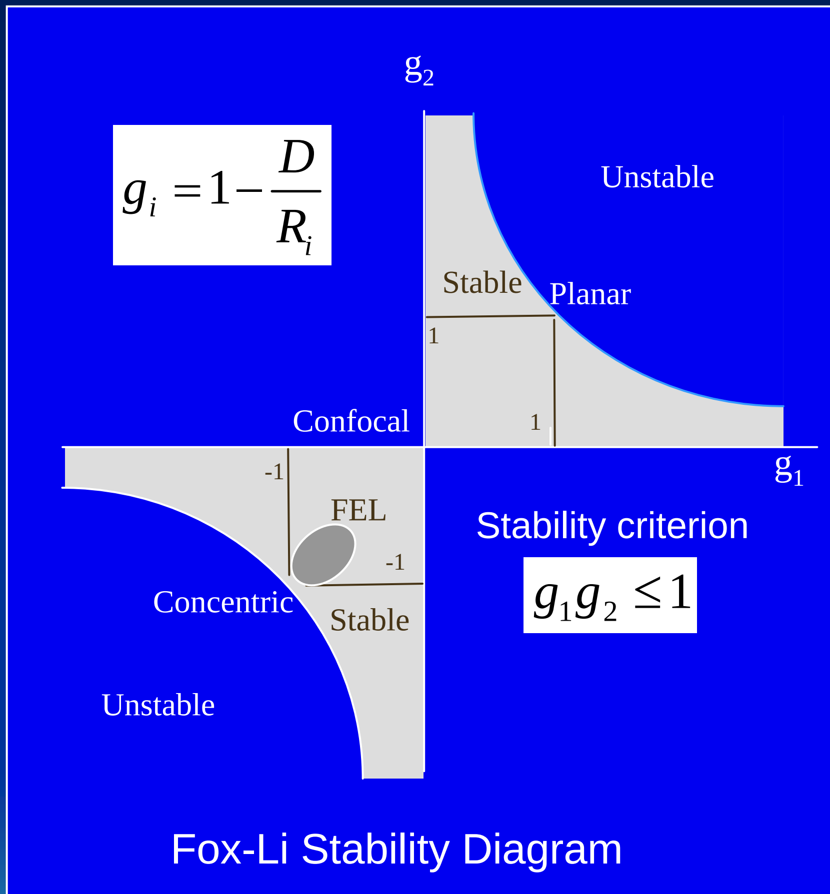

Concentric

Confocal

Common FEL resonators are near-concentric design, D ~ 2R.

The lowest order optical mode of the near-concentric design has small mode area in the wiggler (high optical field) and large mode area at the mirrors, thanks to large divergence angle. The strong divergence reduces FEL intensity on the mirrors and minimizes risk of optical damage.

# Stability, Rayleigh Length and Optical Beam Mode Area



Fox-Li Stability Diagram

Symmetric resonator

$$g_1 = g_2 = g$$

Note: *g* is resonator geometric factor, not FEL gain.

Rayleigh length

$$z_R = R\sqrt{\frac{1+g}{1-g}}$$

Gaussian mode area at waist

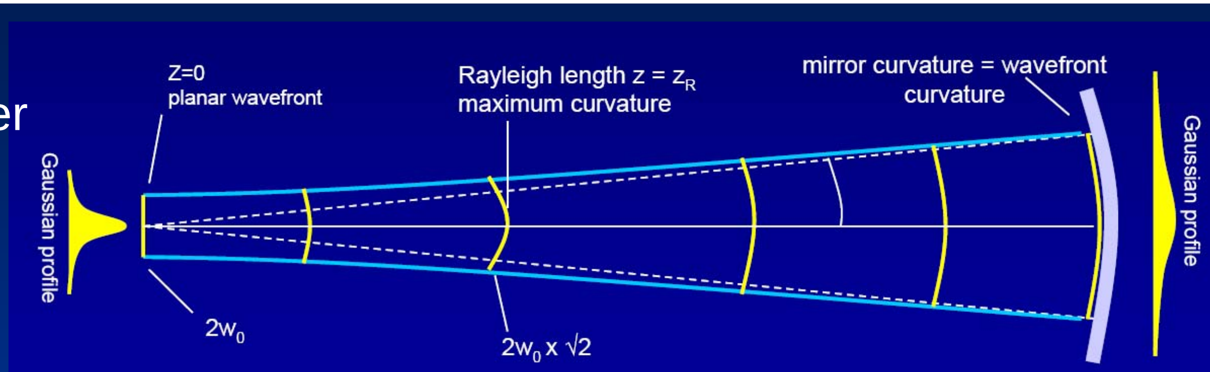
$$\Sigma_0 = D\lambda\sqrt{\frac{1+g}{4(1-g)}}$$

Rayleigh length of a near-concentric cavity is R times square root of (1 + g)/2.

# Optical Beam Mode Areas

Wiggler center

$$\Sigma_0 \sim R\lambda\sqrt{\frac{1+g}{2}}$$



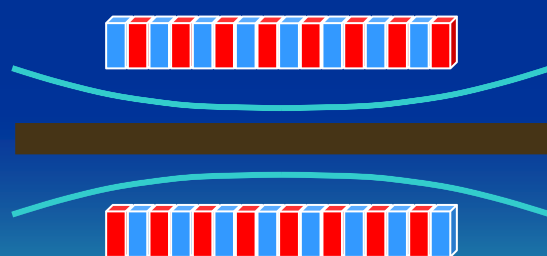

At the mirror

$$\frac{\Sigma_R}{\Sigma_0} = \frac{2}{1+g}$$

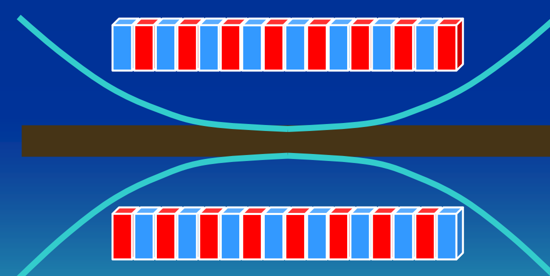
At the entrance and exit of the wiggler $$\frac{\Sigma_w}{\Sigma_0} = 1 + \left(\frac{L_w}{2z_R}\right)^2$$

Optical mode diameters at the wiggler entrance and exit have to be smaller than the wiggler or wiggler tube apertures.

Confocal resonator
Optical beam >> electron beam

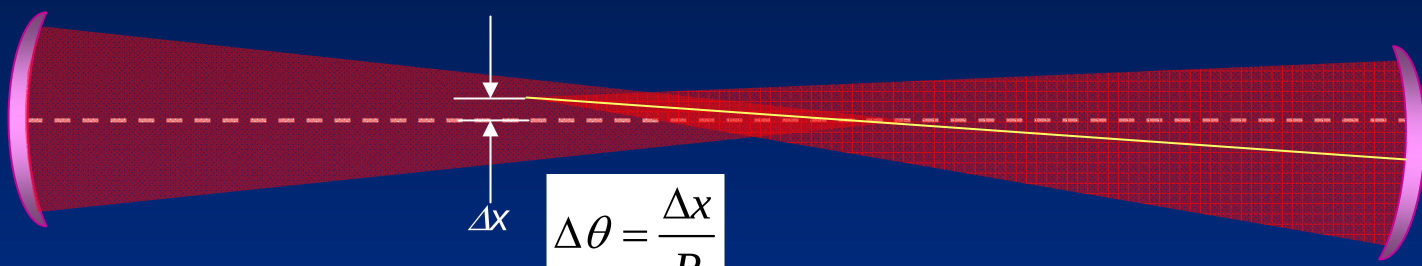


Concentric resonator
Optical beam > electron beam



Short Rayleigh length
Optical beam ~ electron beam

# Angular Sensitivity of Near-concentric Resonators




$$\Delta\theta = \frac{\Delta x}{R}$$

Angular sensitivity for $R_1 = R_2$ ($g_1 = g_2$)

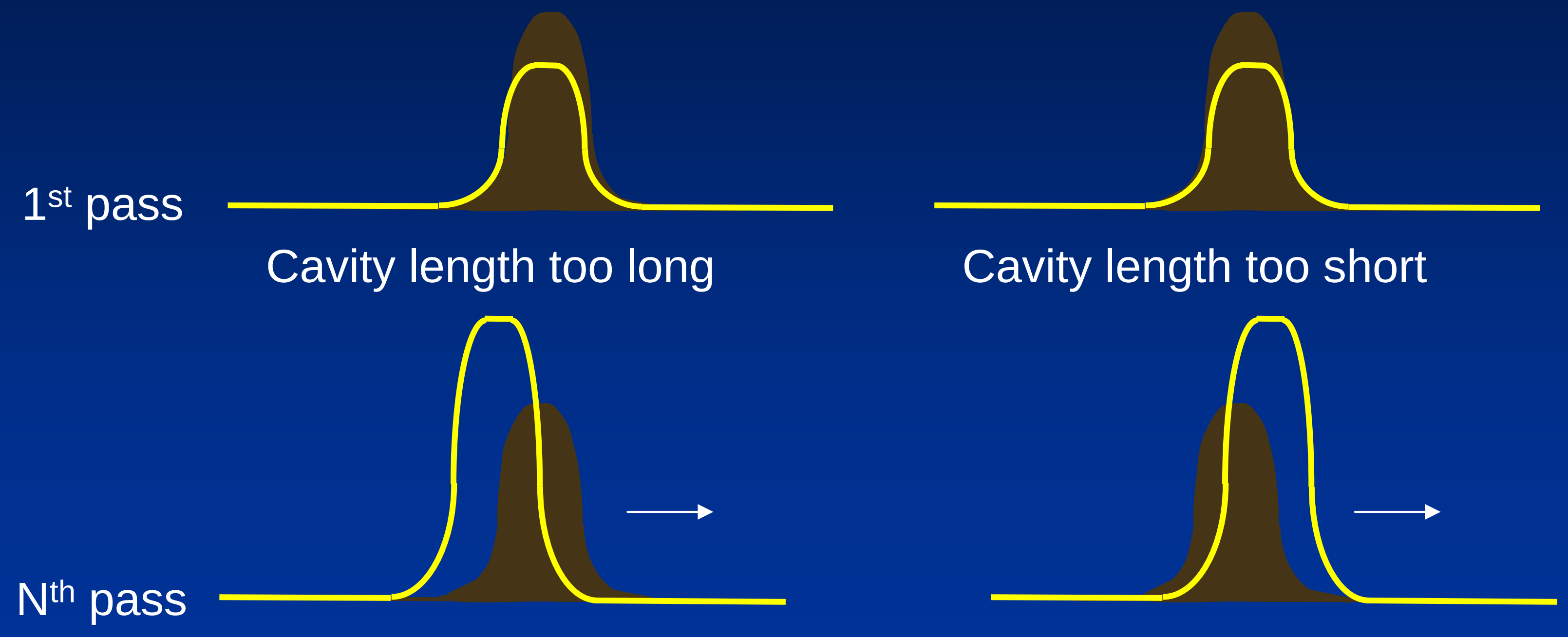
$$\Delta\theta = \sqrt{\frac{\lambda}{\pi D}\left(1-g\right)^{\frac{1}{4}}\left(1+g\right)^{\frac{3}{4}}}$$

For near concentric resonators,
$g = -1 + x$ where $x \ll 1$

$$\Delta\theta \approx \sqrt{\frac{\sqrt{2}\lambda}{\pi D} x^{\frac{3}{4}}}$$

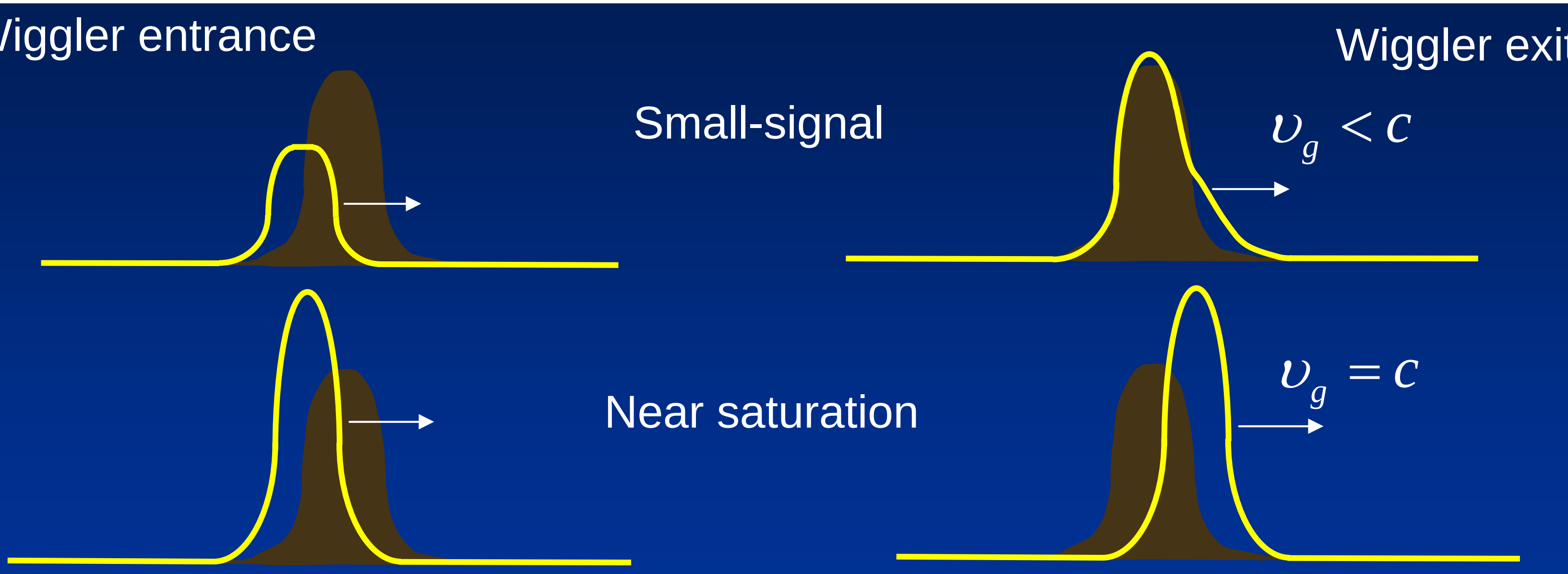
Example: For an FEL with near-concentric resonator with $g = 0.98$ ($x = 0.02$), with $\lambda = 1\ \mu$ and D = 20 m, the mirror angular sensitivity $\Delta\theta$ is 11 $\mu$rad.

# Optical Pulse & Cavity Length




- Cavity length must be such that the optical pulses overlap with the electron bunches in N passes in order to reach saturation (N could be in the hundreds or in the thousands).
- Mirror separation must be equal to the spacing between electron bunches to an accuracy of the FEL detuning length (about 2 - 3 wavelengths).

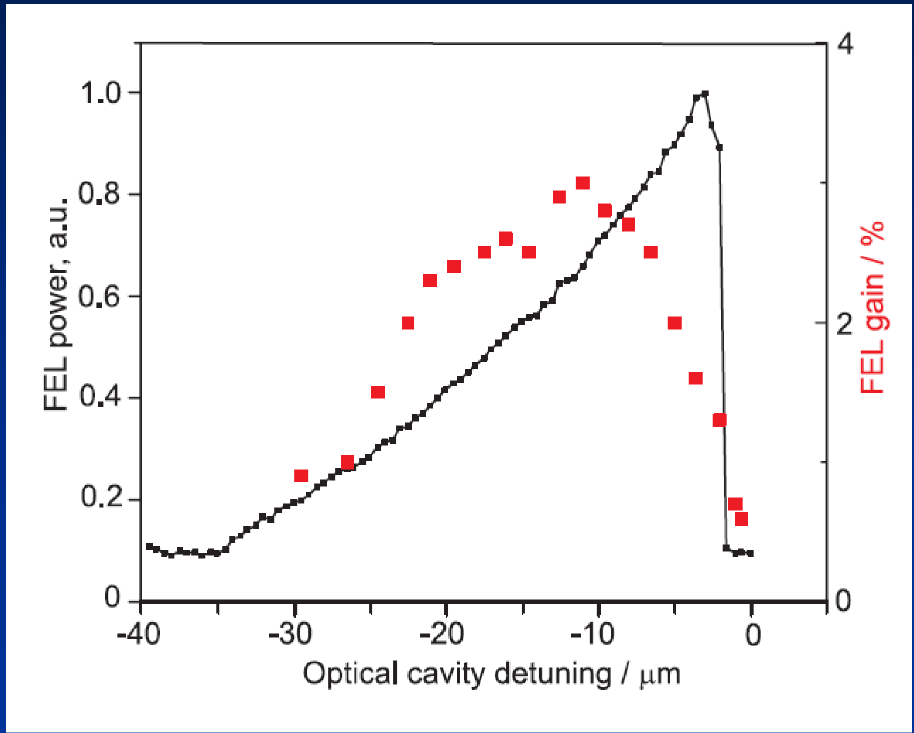
# Slippage & Lethargy




- In the small-signal regime, the trailing edge is amplified more than the leading edge. The group velocity of the optical pulse is less than *c*. Gain is maximized at slightly shorter cavity length.
- Near saturation, gain is reduced and the optical pulse slips ahead by $N_w\lambda$ (the slippage length). Power is maximized at a cavity length close to the electron bunch spacing.

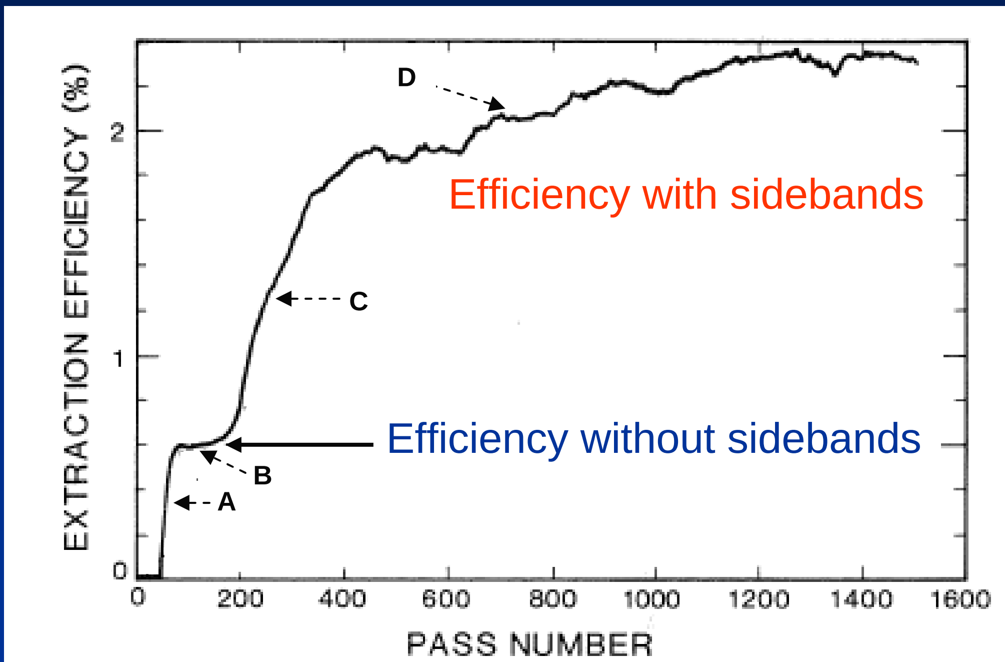
Slippage length $l_s = N_w \lambda$

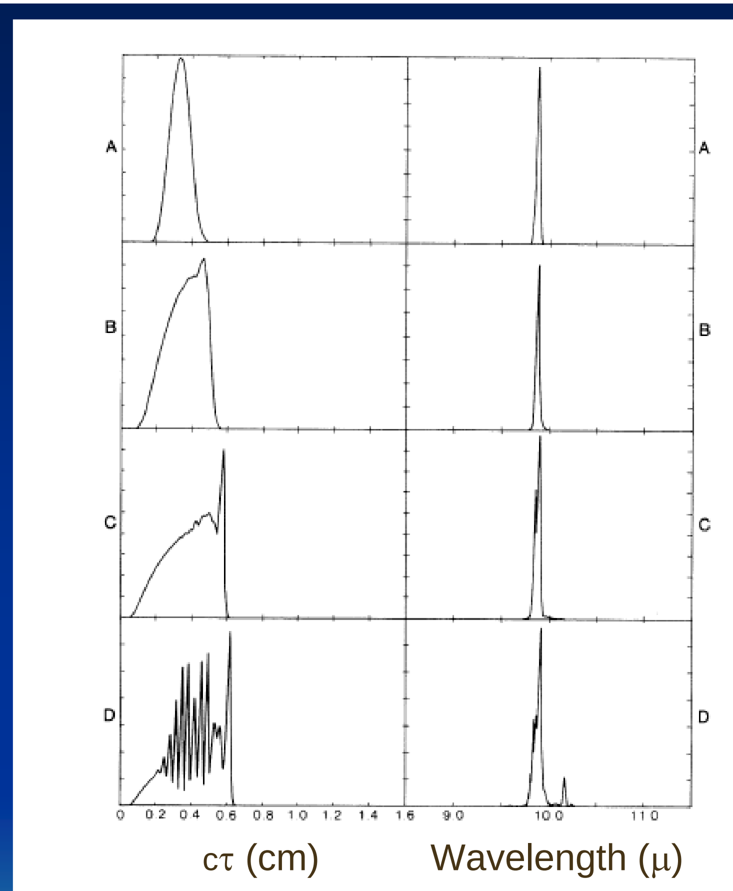
# Cavity Length Detuning




Saturation power (black line) and FEL net gain (red dots) versus cavity length detuning for the ELBE oscillator FEL experiment at 20 μ. The cavity length must be initially shortened to maximize gain. After oscillation is established, the cavity is lengthened to maximize output power. When the cavity length exceeds the electron bunch spacing (*c* divided by *f*), FEL power drops to zero.

# Sideband Generation

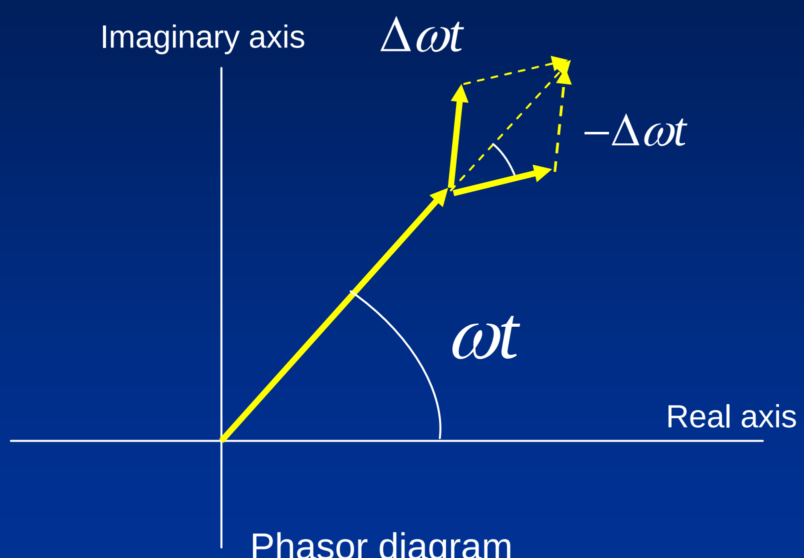


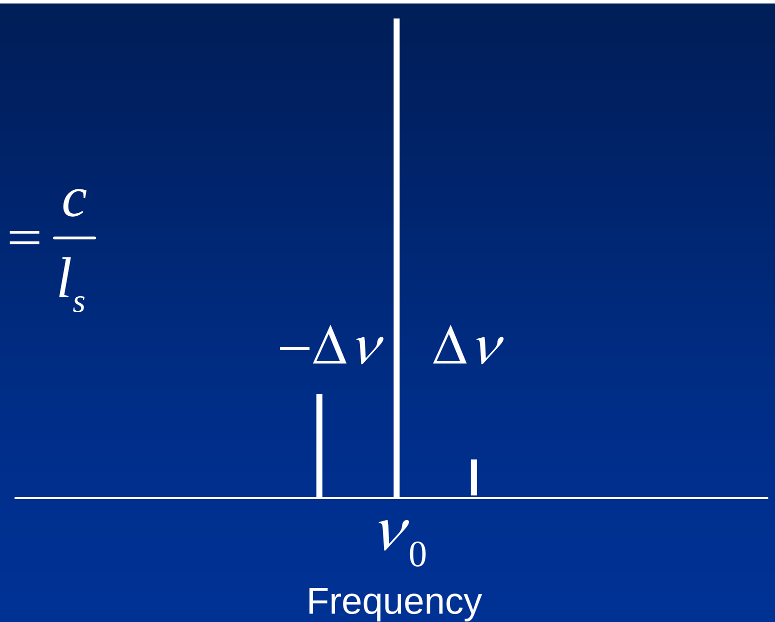

Sidebands are generated at zero cavity detuning after the FEL saturates. The FEL pulses develop ultrashort spikes and its spectrum shows additional peaks (sidebands)

FEL efficiency can be increased by a factor of 2-4X with sideband generation.

# Phasor Diagram of Sidebands




$$\Delta\nu = \frac{c}{l_s}$$



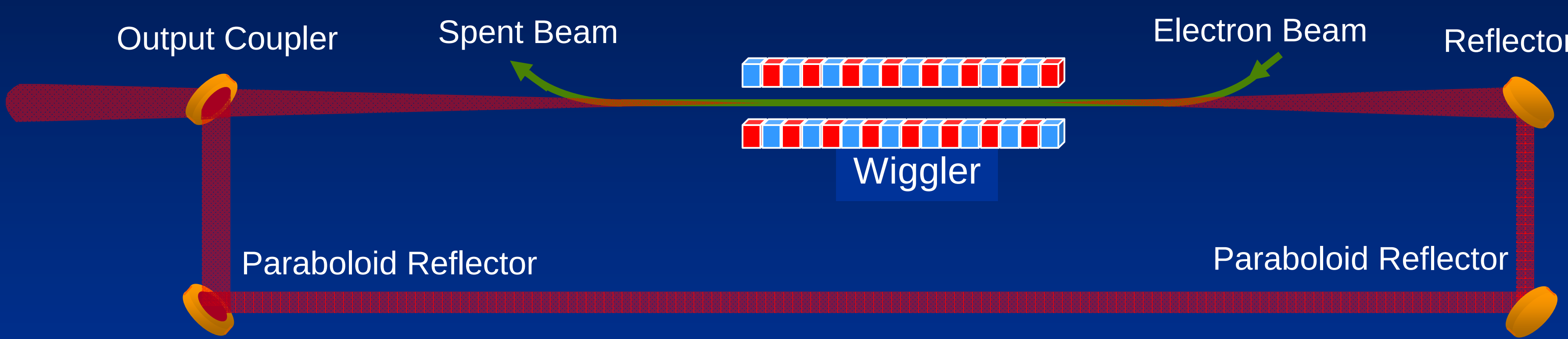

At saturation, synchrotron oscillation causes the optical intensity to be modulated in time. This modulation in turn causes the electron motions in phase space to be modulated. The FEL micropulses break up into individual spikes whose width is on the order of a slippage length. Sidebands are spectrally separated from the central frequency by *c* divided slippage length. The low-frequency sideband is more intense than the high-frequency one.

# Regenerative Amplifier FEL

Regenerative Amplifier FEL (RAFEL) uses a combination of high-current electron beam and a **low-Q optical resonator** to reach saturation in a few passes through the wiggler. Each pass involves a new electron bunch interacting with the optical field and bunching near the end of the wiggler. The optical intensity grows exponentially (high gain) in each pass until it reaches the saturation intensity. Exponential gain depends on emittance, energy spread and peak current. At saturation FEL power oscillates between high (optimum feedback) and low (over-bunched).

RAFEL optical feedback varies depending on **optical guiding** in the wiggler. The FEL optical mode is determined mainly by the electron beam. The resonator cavity length detuning is a fraction of the electron bunch length. Maximum power occurs on both sides of zero detuning length.

# Regenerative Amplifier Feedback




Schematic of the Regenerative Amplifier FEL

Gain per pass

$$G_{ss} = \frac{1}{9}\exp\left[\frac{L_w}{L_G}\right]$$

Optical feedback

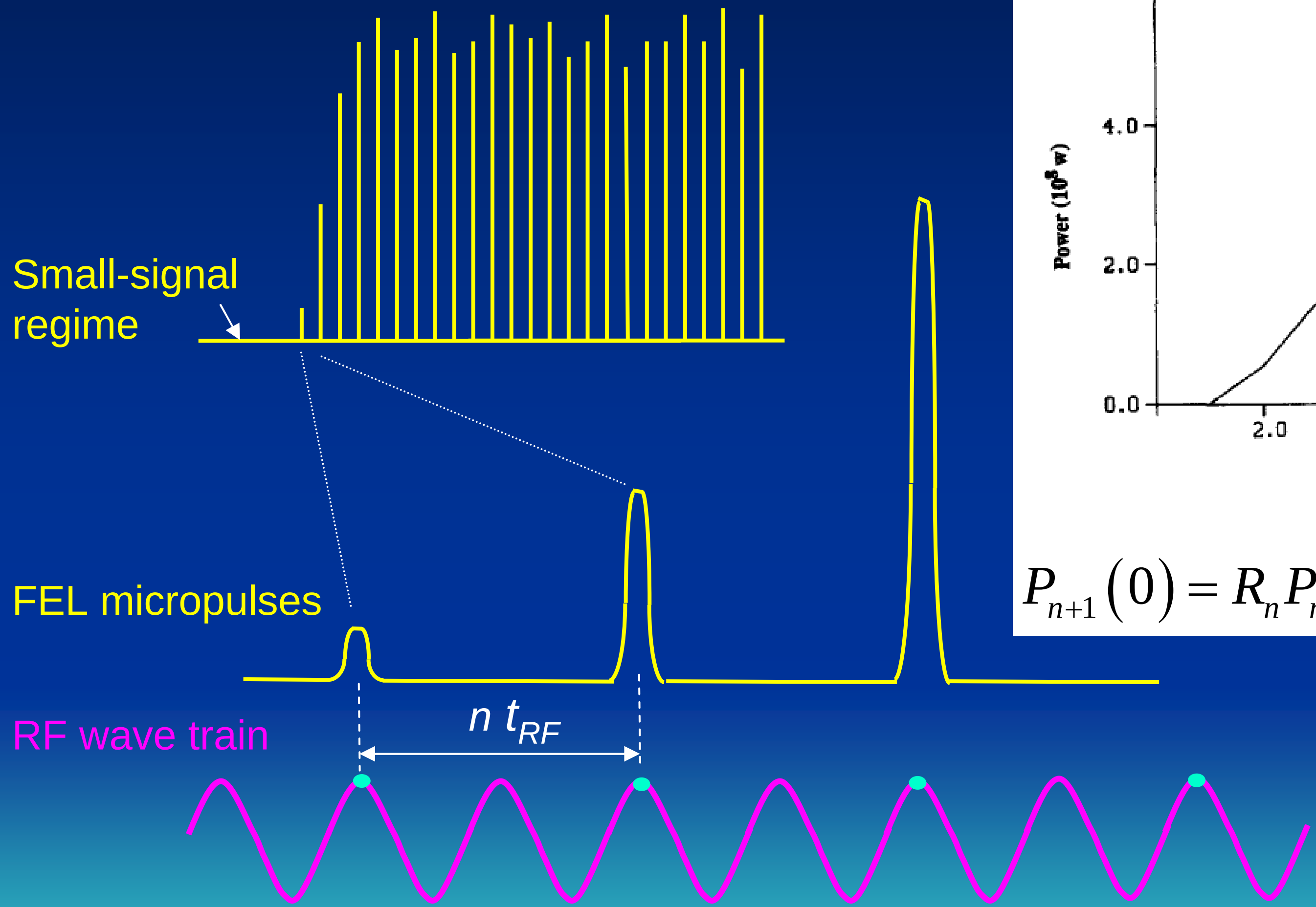
$$R_{FB} = R_{OC} R_M^3 f_m$$

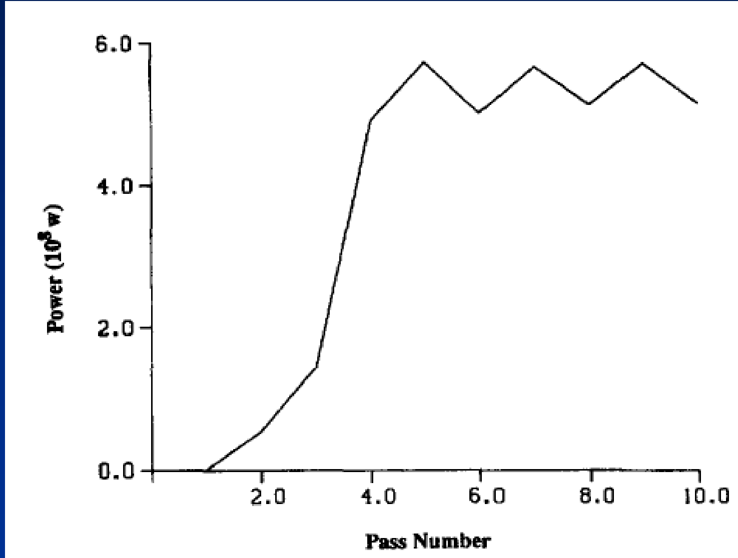
Net gain

$$G_{net} = R_{FB} G_{ss}$$

Optical feedback is less than the reflectivity of mirrors because only a small fraction, $f_M$ of the returning optical beam matches with the guided mode that can be amplified in the next pass.

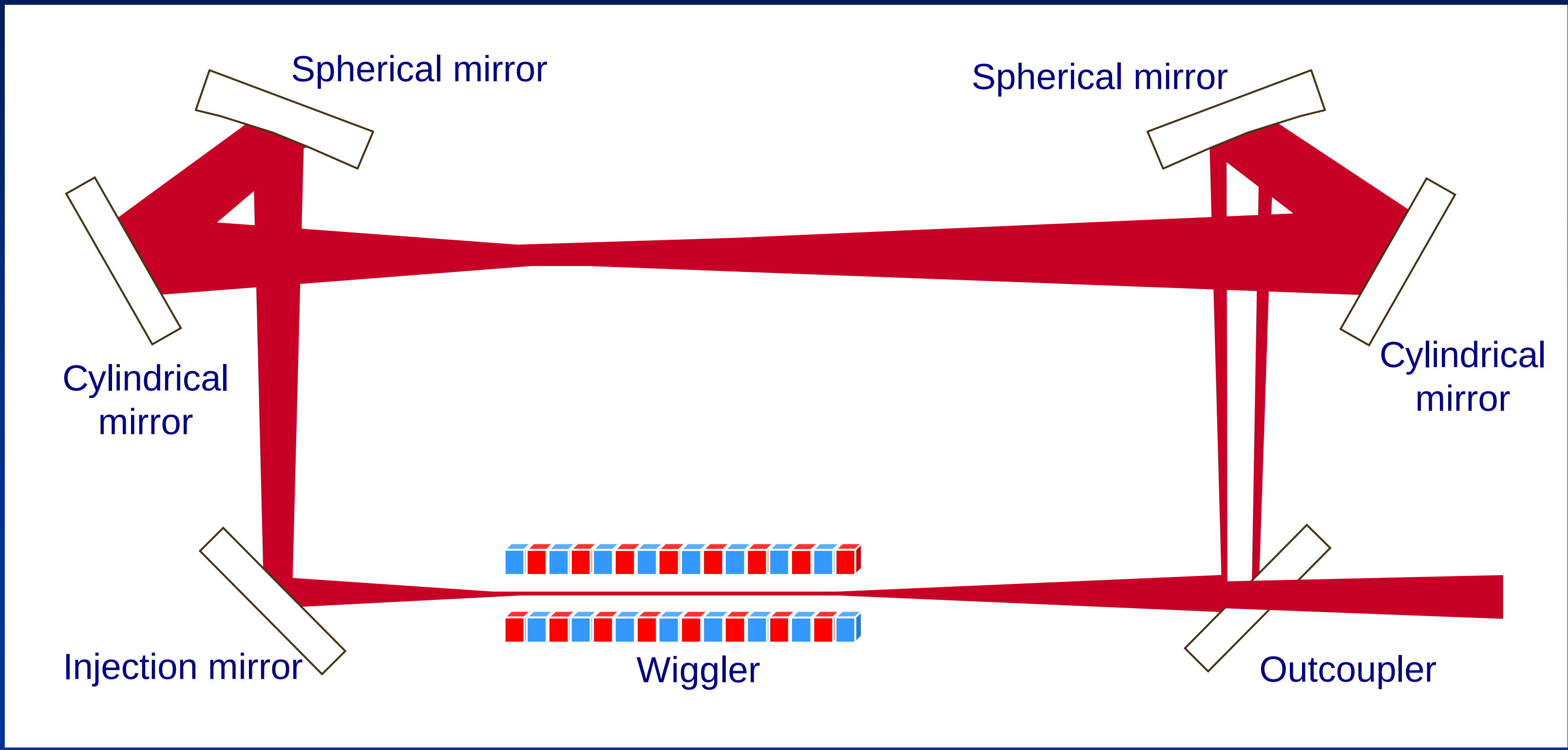
# RAFEL Power Growth per Pass



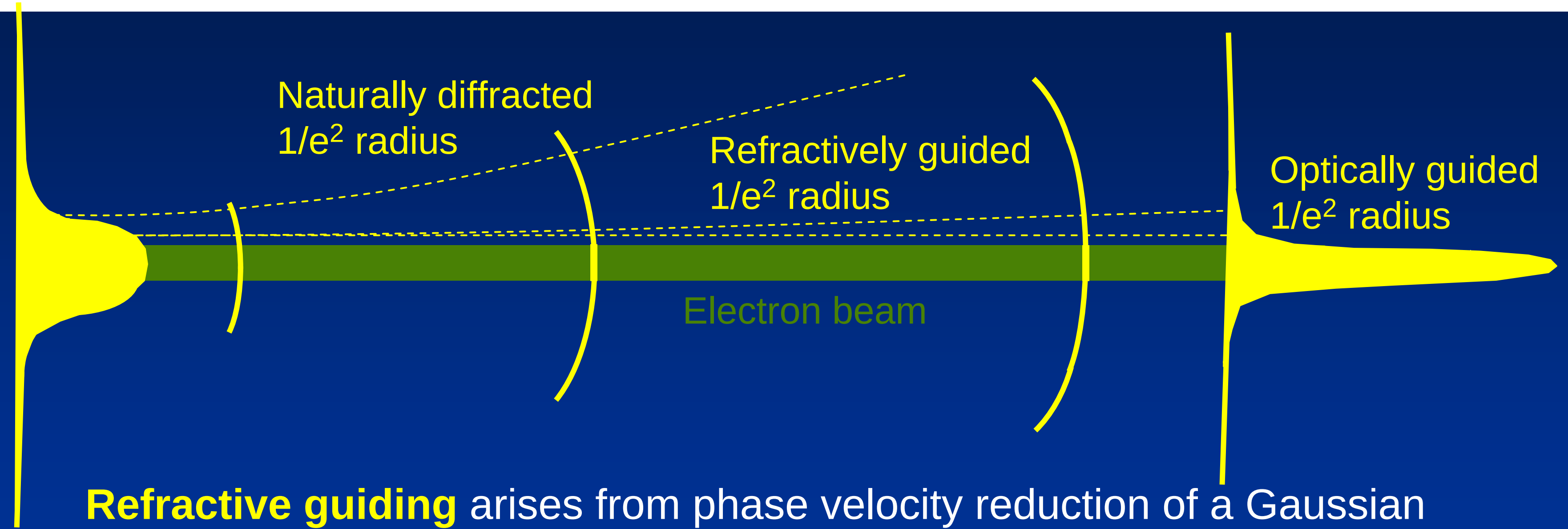

$$P_{n+1}(0) = R_n P_n(L_w) = R_n P_n(0) e^{\frac{L_w}{L_g}}$$

# Optical Feedback Cavity




Ring cavity length is set to be synchronous with electron bunch arrival. Optical mode is guided by the electron beam in the wiggler. Without optical guiding, the optical mode is not reproduced from pass to pass.

# Optical Guiding in FEL




**Refractive guiding** arises from phase velocity reduction of a Gaussian beam near the center of the electron beam. Refractive guiding depends on the real part of the complex refractive index.

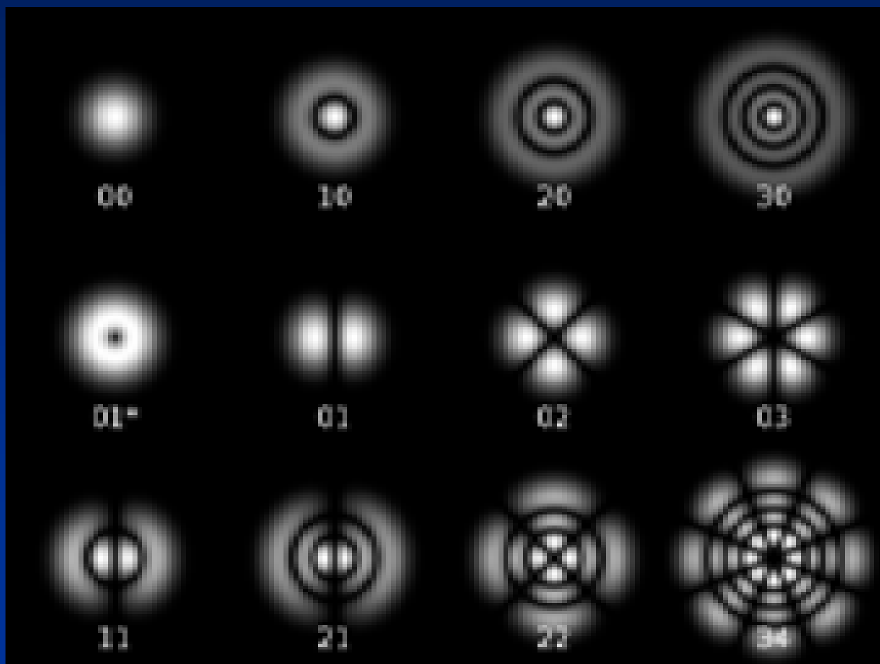
**Gain guiding** is due to selective amplification of the central part of the optical beam that overlaps with the high-current electron beam. Gain guiding depends on the imaginary part of the complex refractive index.

The combination of diffraction and gain guiding leads to mode selection.

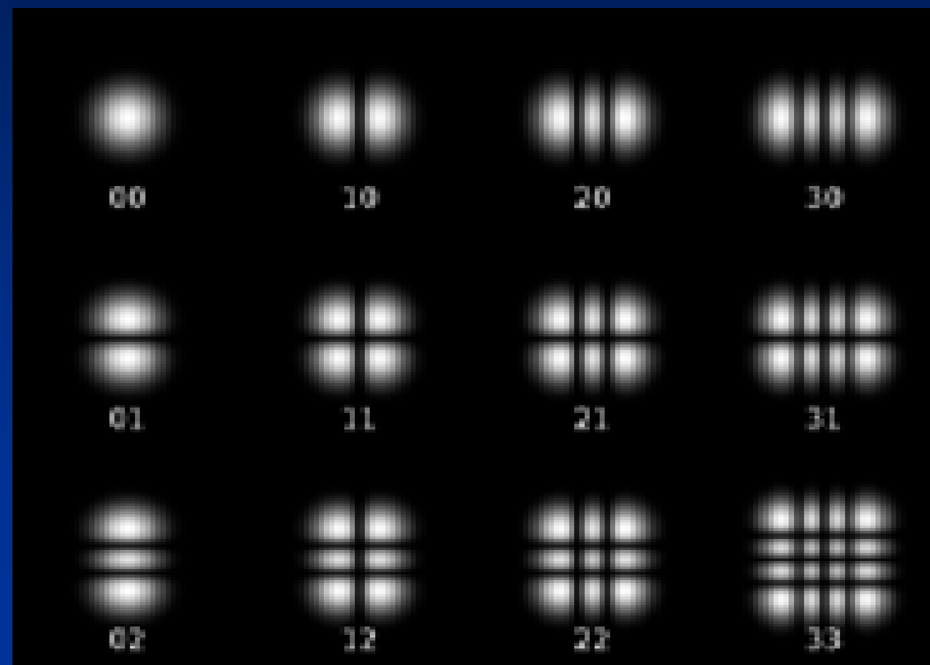
# Optical Modes

Cylindrical (ρ θ) Coordinates

Cartesian (x y) Coordinates




Laguerre-Gaussian (LG) Modes




Hermite-Gaussian (HG) Modes

The lowest order mode ($LG_{00}$ or $HG_{00}$) has the strongest on-axis intensity to interact with the electron beam, and thus sees the highest FEL gain.

# Electron Beam's Refractive Index

Electron beam's complex refractive index

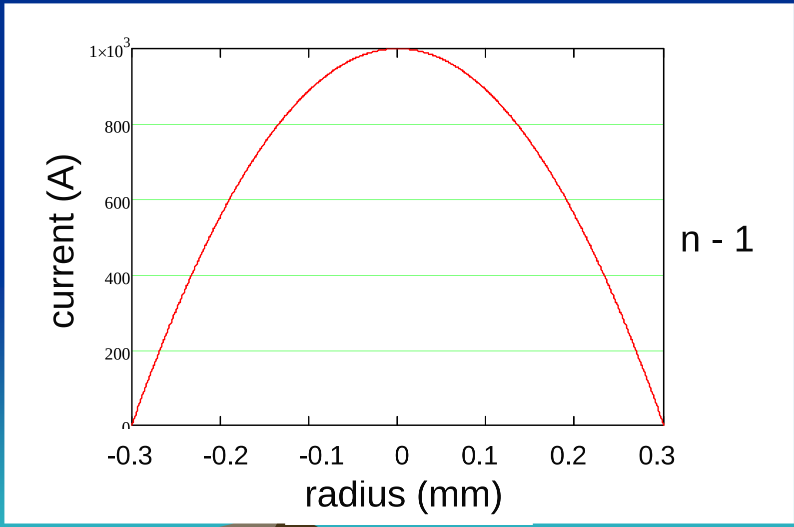
-Imaginary term = gain

$$-\mathrm{Im}(n) = \frac{1}{k}\frac{da_s}{dz} \propto \left(\frac{I}{I_A}\right)\left\langle\frac{\sin\theta}{\gamma}\right\rangle$$

Real term = refractive focusing

$$\mathrm{Re}(n) - 1 = \frac{1}{k}\frac{d\phi_s}{dz} \propto \left(\frac{I}{I_A}\right)\left\langle\frac{\cos\theta}{\gamma}\right\rangle$$

Electrons tend to bunch around $\theta$ slightly positive of 0, thus the $\sin\theta$ (gain) term is positive and the $1 + \cos\theta$ (refractive index) term is slightly larger than unity. The FEL optical beam sees a refractive index difference from vacuum, $n - 1$, that is proportional to electron beam peak current. This refractive index is higher at the center of the electron beam and lower at the edge, similar to an optical fiber.



# Optical Beam Guided Mode Radius

$$\frac{d^2r}{dz^2} - \frac{\lambda^2}{\pi^2 r^3} + \frac{\lambda r}{\pi\sqrt{3}L_G\sigma_b^2} = 0$$

Natural diffraction

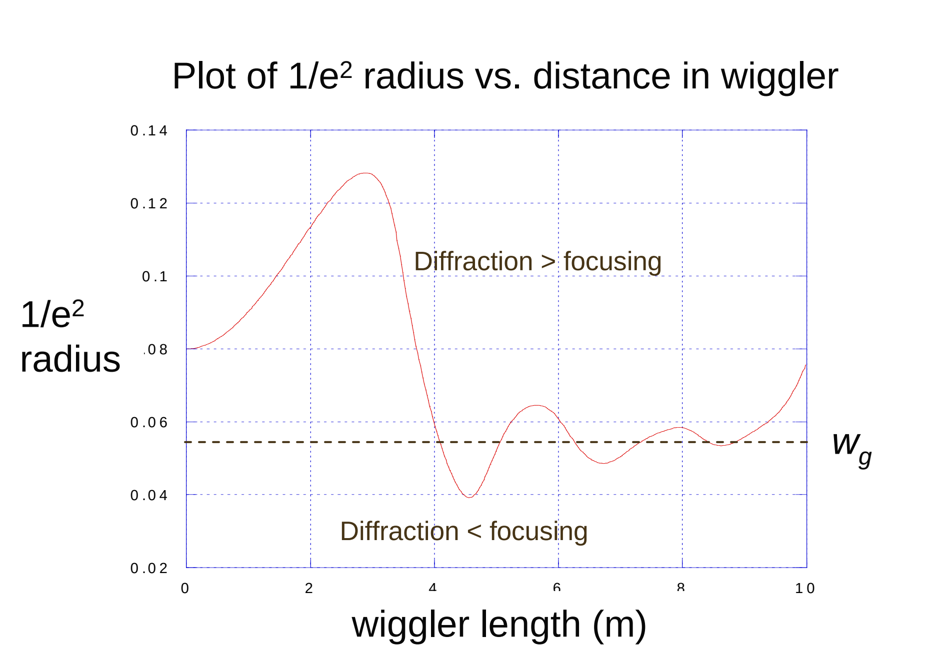
Focusing from gain/refraction

Guided mode radius

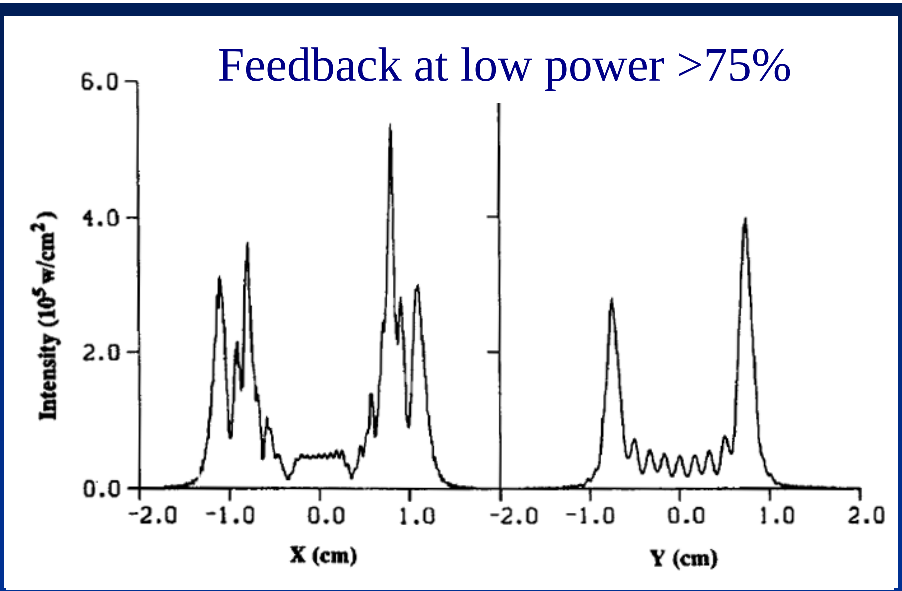
$$\boxed{r_g = \left(\frac{\sqrt{3}\lambda L_G\sigma_b^2}{\pi}\right)^{1/4}}$$

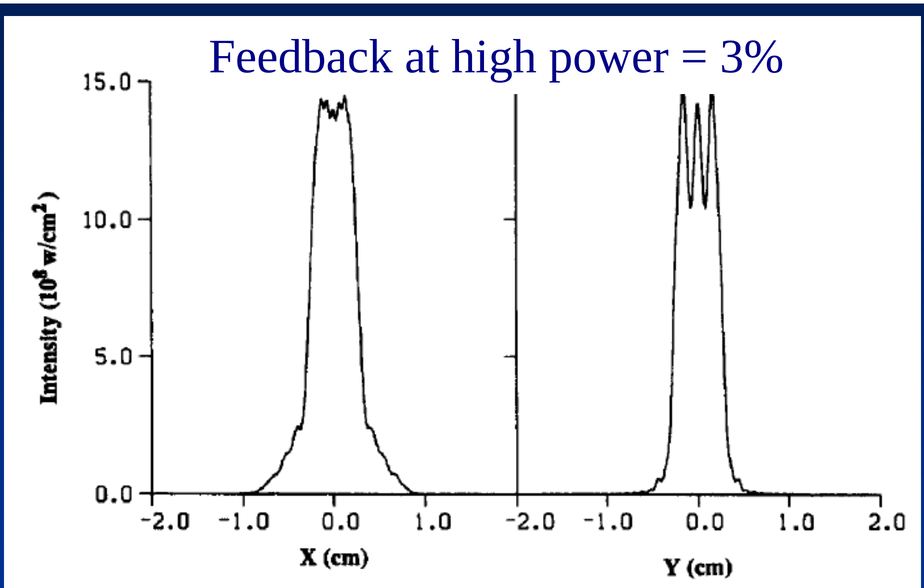
Guided 1/e² mode radius in 1D

$$\boxed{w_g = \left(\frac{2\lambda_w^2\sigma_b^2}{\gamma^2\pi^2\rho}\left(1+a_w^2\right)\right)^{1/4}}$$



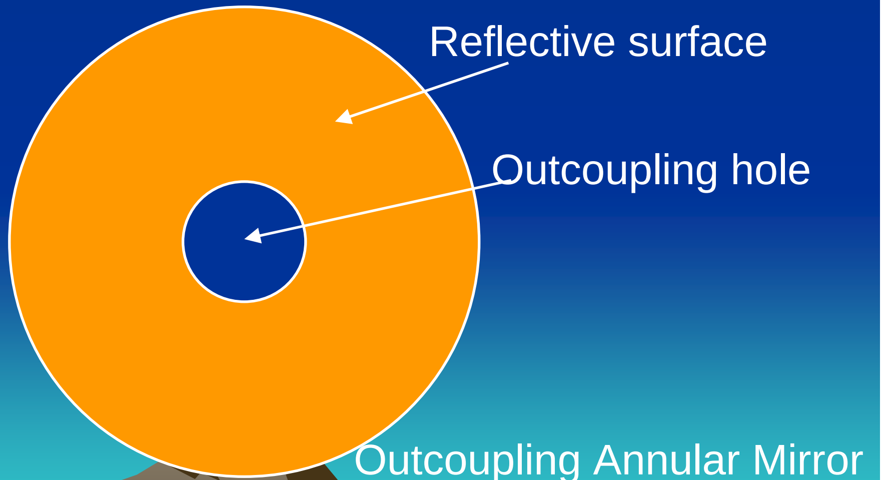
# Variable Feedback in RAFEL



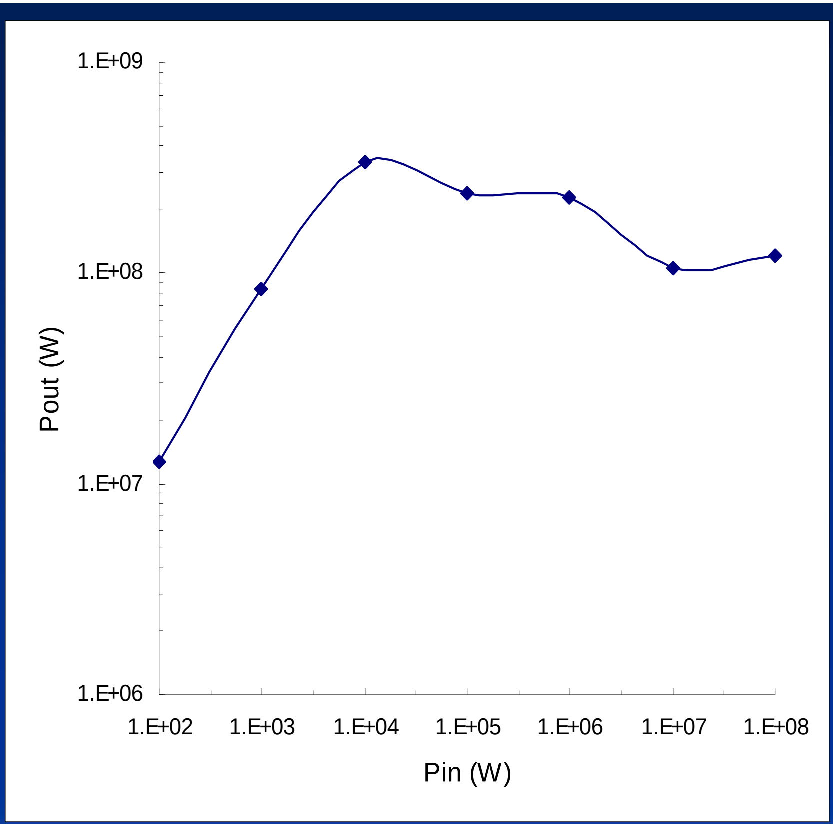



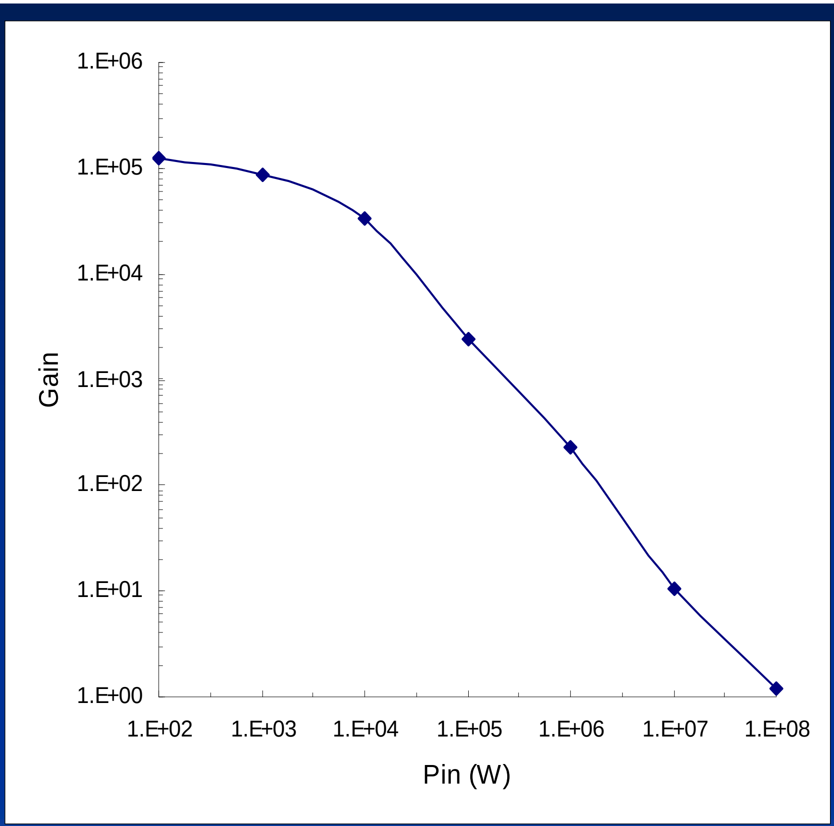

Optical feedback is large at small signals to provide high net gain

Optical feedback is reduced at high intensity and most of FEL power exits through the large hole on the annular mirror.



# Output Power, Gain vs Input Power



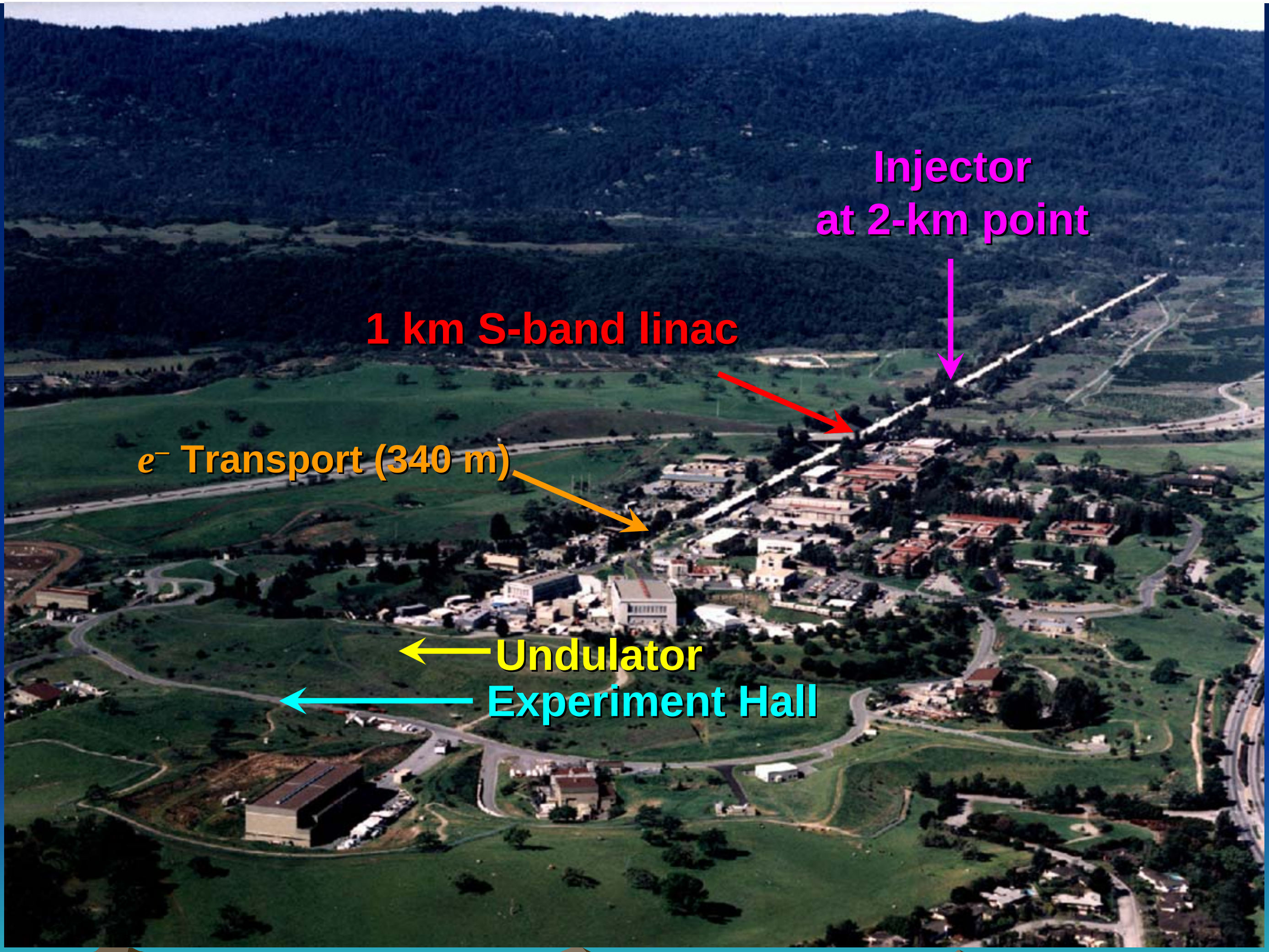




Maximum power and maximum gain occur at different input power. Maximum gain occurs at low $P_{in}$. Maximum power occurs at intermediate $P_{in}$. Output power oscillates between high and low because $P_{in}$ varies from pass to pass.

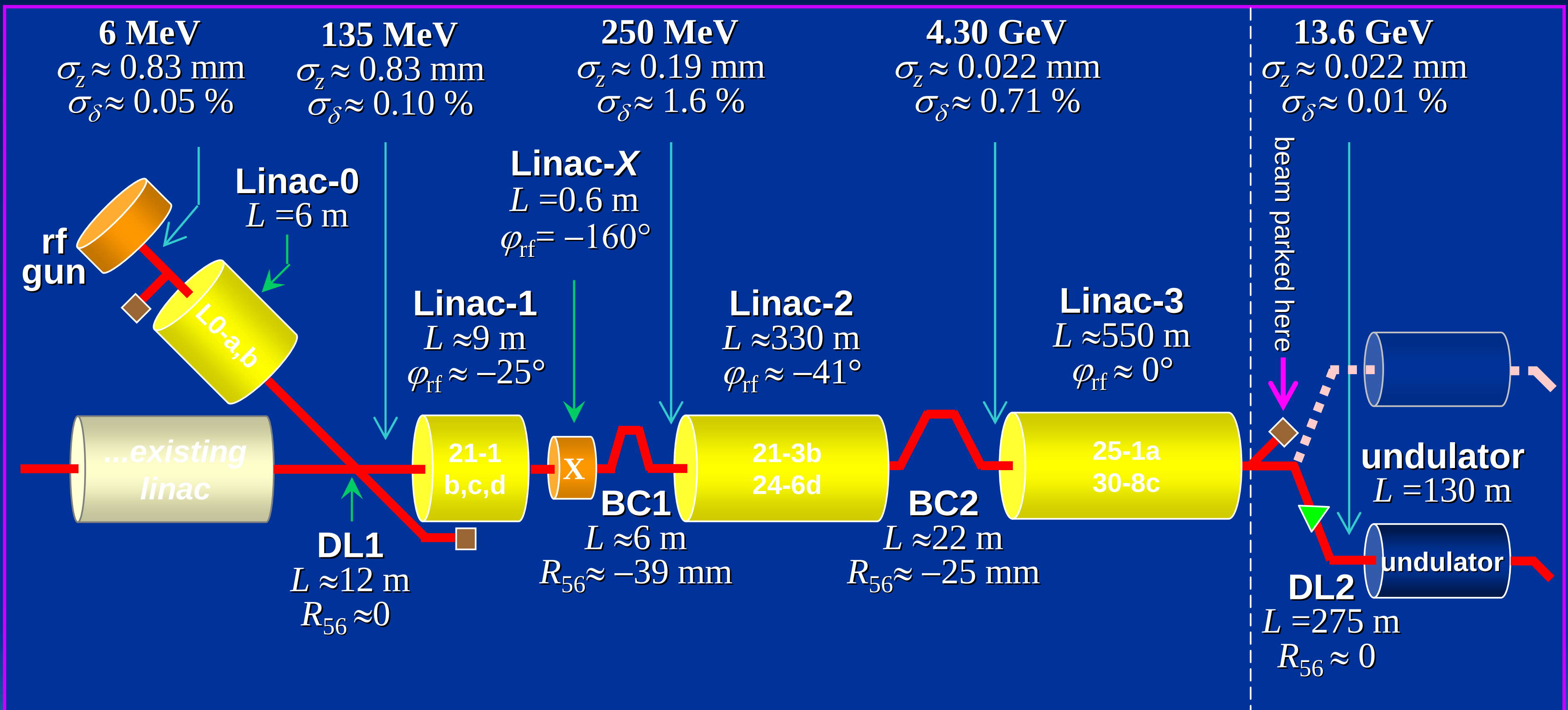
# Self-Amplified Spontaneous Emission

SASE FEL uses a combination of high-current electron beam and a long wiggler (undulator) to reach saturation in a single pass. SASE starts up from the random positions of electrons in the bunch, i.e. **noise**. The initial interaction is linear, that is, electrons outside the separatrix interacting with the optical field become bunched and amplify the optical field. SASE power grows exponentially (**exponential gain**) along the wiggler with a characteristic e-folding length called the power gain length. At higher intensity, the electrons become trapped in the growing bucket and the FEL interaction becomes non-linear. Saturation occurs when the over-bunched electrons at the bottom at the bucket rotate upward. SASE extraction efficiency is approximately the same as the gain coefficient $\rho$. SASE is the baseline design for many **x-ray FEL** around the world, since no materials exist with sufficient reflectivity as x-ray mirrors. SASE FEL is fully **coherent transversely** and only **partially coherent longitudinally**. The spectral bandwidth is narrow in the exponential gain region but broadens at saturation. Shot-to-shot energy fluctuations are large in the exponential gain region.

# Linac Coherent Light Source

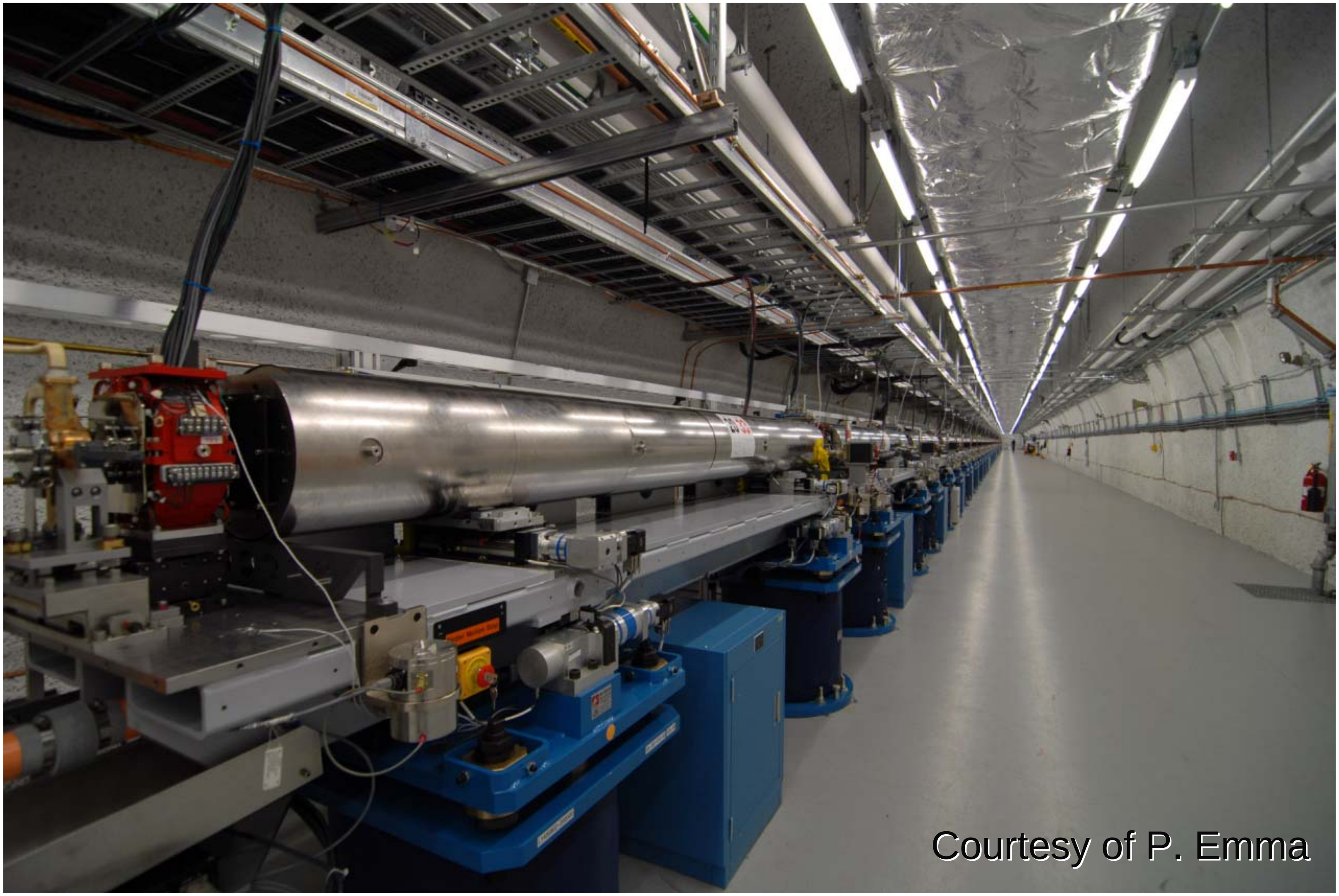


# LCLS Accelerator Layout



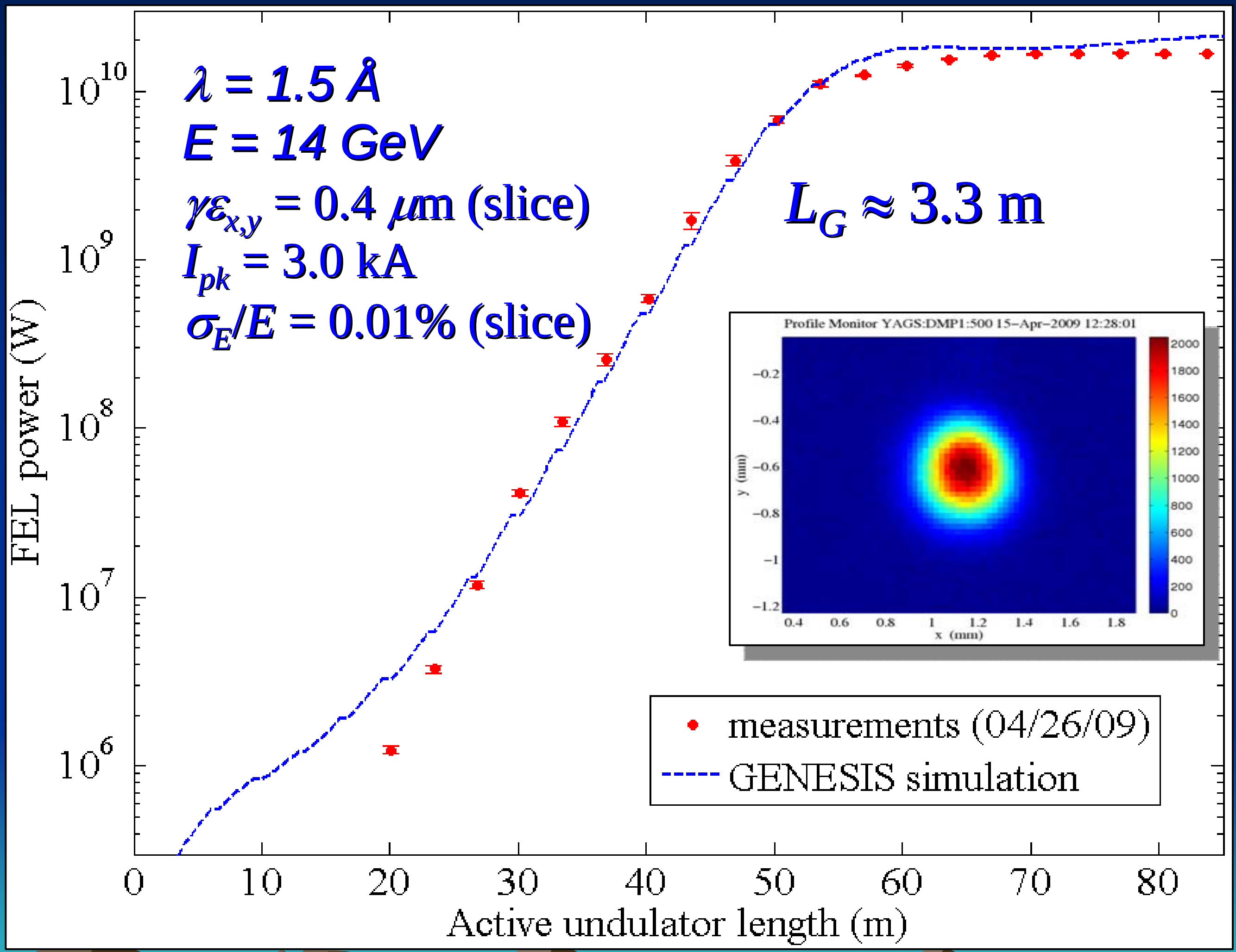

Courtesy of P. Emma

# LCLS Undulator Hall



Courtesy of P. Emma

# LCLS Power Growth




Courtesy of P. Emma

# Start-up Power

Spectral power grows exponentially with z from start-up power

$$\frac{dP}{d\omega} = \left( \left.\frac{dP}{d\omega}\right|_{seed} + \left.\frac{dP}{d\omega}\right|_{startup} \right) e^{-\left(\frac{\Delta\omega}{\sigma_\omega}\right)^2} e^{\frac{z}{L_G}}$$

Start-up power without a seed laser depends on the number of correlated electrons, characterized by the initial bunching coefficient, and the resulting equivalent start-up power.

$$b_1 = \frac{1}{N_e} \sum_{k=1}^{N_e} e^{i\omega t_k(0)}$$

$$P_{startup} = \left| b_1 \right|^2 N_e^2 P_e$$

For SASE, the initial bunching is due to shot noise so the square of $b_1$ is simply $1/N_e$ and the SASE start-up is spontaneous emission. For pre-bunched electrons, the start-up power is

$$P_{prebunch} = 0.22 \left| b_1 \right|^2 \rho \left( \frac{IE_b}{e} \right)$$

# Exponential Gain Length

1D Power gain length

$$L_G = \frac{\lambda_w}{4\pi\sqrt{3}\rho}$$

Wiggler length over which power grows by a factor of 2.718.

1D Field gain length

$$L_g = \frac{\lambda_w}{2\pi\sqrt{3}\rho}$$

Wiggler length over which field grows by a factor of 2.718 (twice as long as power gain length).

Requirements for SASE

Normalized emittance $\varepsilon_n \le \frac{\gamma\lambda}{4\pi}$

Energy spread $\frac{\delta\gamma}{\gamma} \le \rho$

Diffraction $L_G \le Z_R$

**Gain length will increase if any of the above requirements is not met.**

# Three-Dimensional Effects

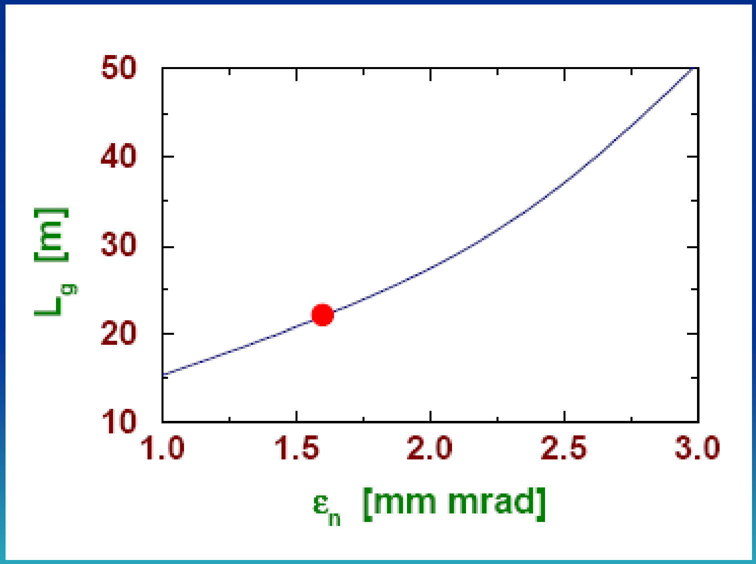
First calculate 1D gain length, then use M. Xie's parameterization to calculate 3D gain length, longer than 1D gain length due to 3D effects.

$$\boxed{L_{3D} = L_{1D}\left(1+\Lambda\right)}$$

$$\boxed{\begin{aligned}\Lambda = {} & 0.45\eta_d^{0.57} + 0.55\eta_\varepsilon^{1.6} + 3\eta_\gamma^{2} + 0.35\eta_\varepsilon^{2.9}\eta_\gamma^{2.4} + 51\eta_d^{0.95}\eta_\gamma^{3} + 0.62\eta_d^{0.99}\eta_\varepsilon^{1.1} \\ & + 5.3\eta_d^{0.76}\eta_\varepsilon^{2.3}\eta_\gamma^{2.7} + 120\eta_d^{2.1}\eta_\varepsilon^{2.9}\eta_\gamma^{2.8} + 3.7\eta_d^{0.43}\eta_\varepsilon\eta_\gamma\end{aligned}}$$

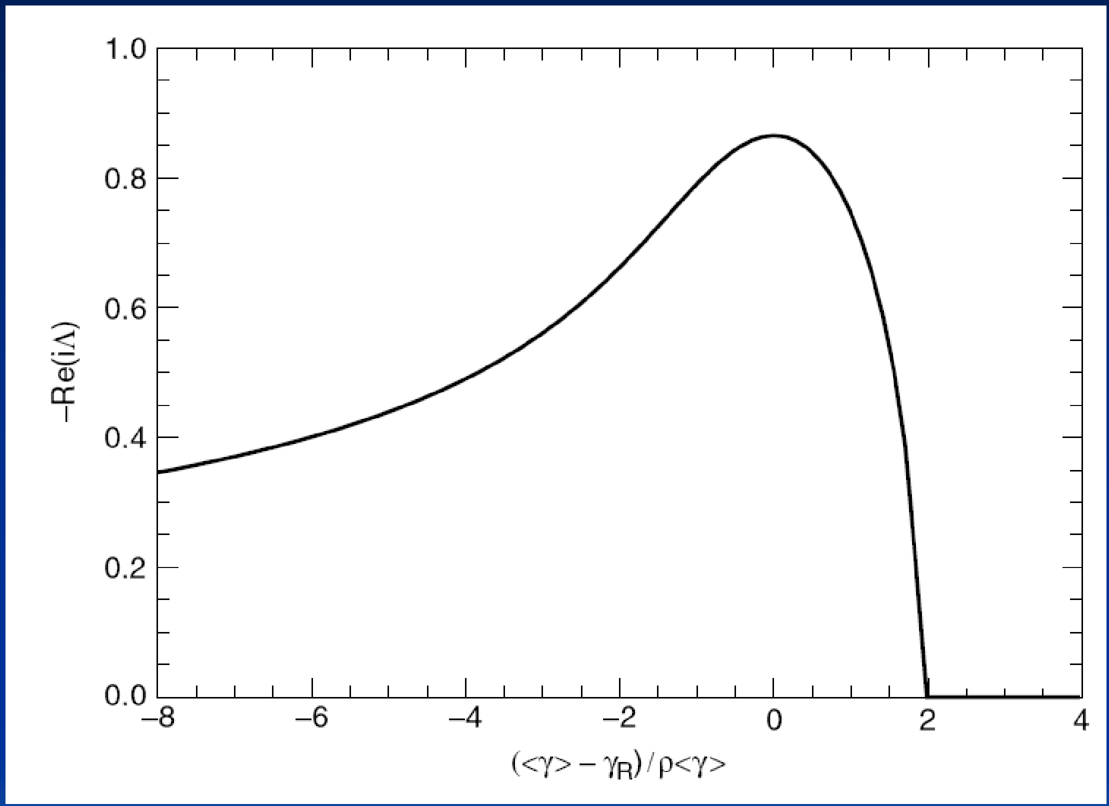
- Diffraction $\eta_d = \dfrac{L_{1D}}{Z_R}$
- Emittance $\eta_\varepsilon = \left(\dfrac{L_{1D}4\pi}{\beta_b\gamma\lambda}\right)\varepsilon_n$
- Energy spread $\eta_\gamma = \left(\dfrac{L_{1D}4\pi}{\lambda_w}\right)\dfrac{\delta\gamma}{\gamma}$

European XFEL gain length vs. emittance



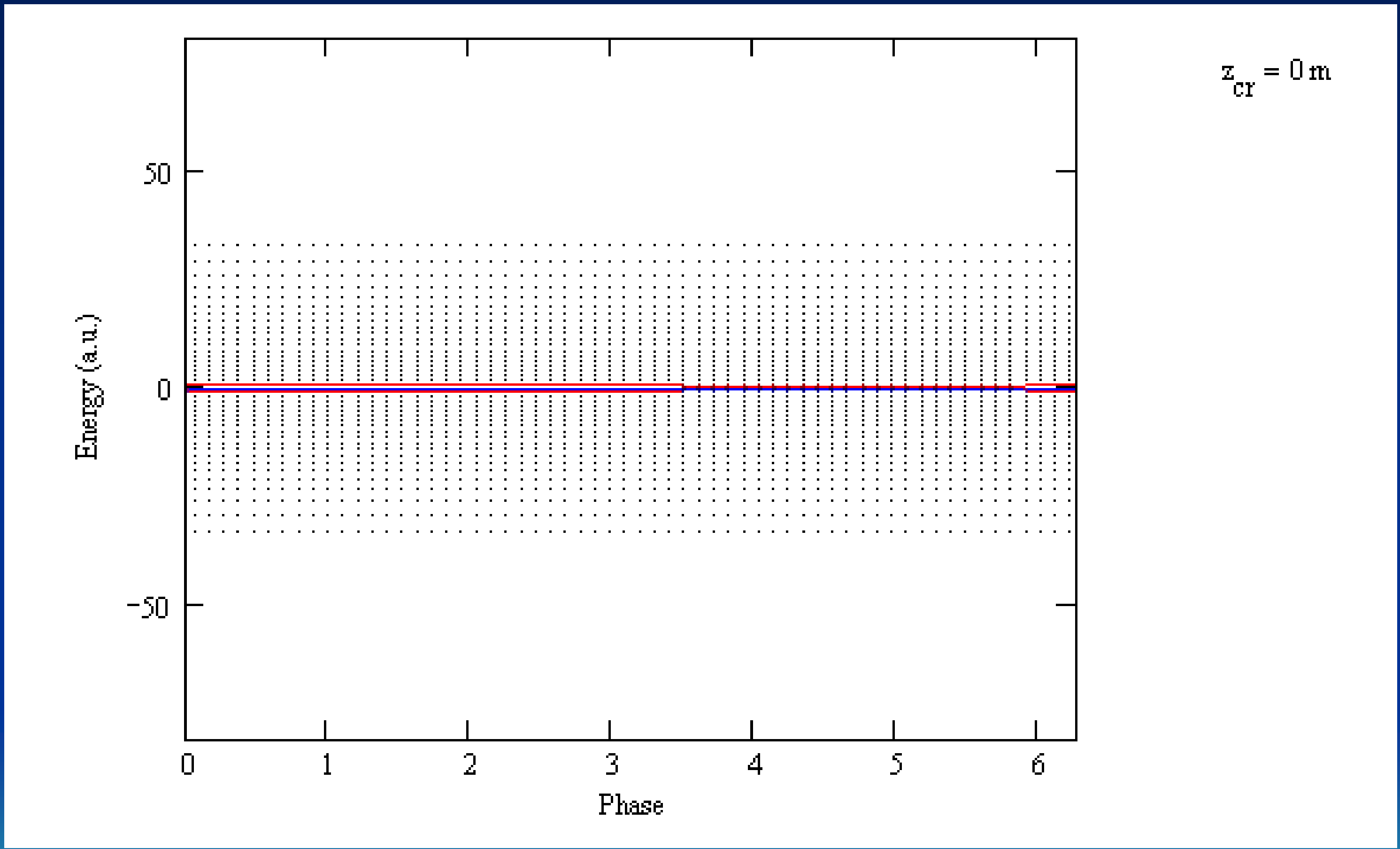
# Gain Spectrum of High-Gain SASE

(a)




Unlike the low-gain FEL which has zero gain at resonance, high-gain SASE FEL has maximum gain at zero detuning (resonance energy).

# SASE Animation



# Rate of Energy Loss and Δγ/γ Increase

- Rate of energy loss due to FEL interaction

$$\frac{dE_b}{dz} = \frac{2r_e\gamma^2 B_w^2}{3}$$

- Increase in energy spread due to FEL interaction

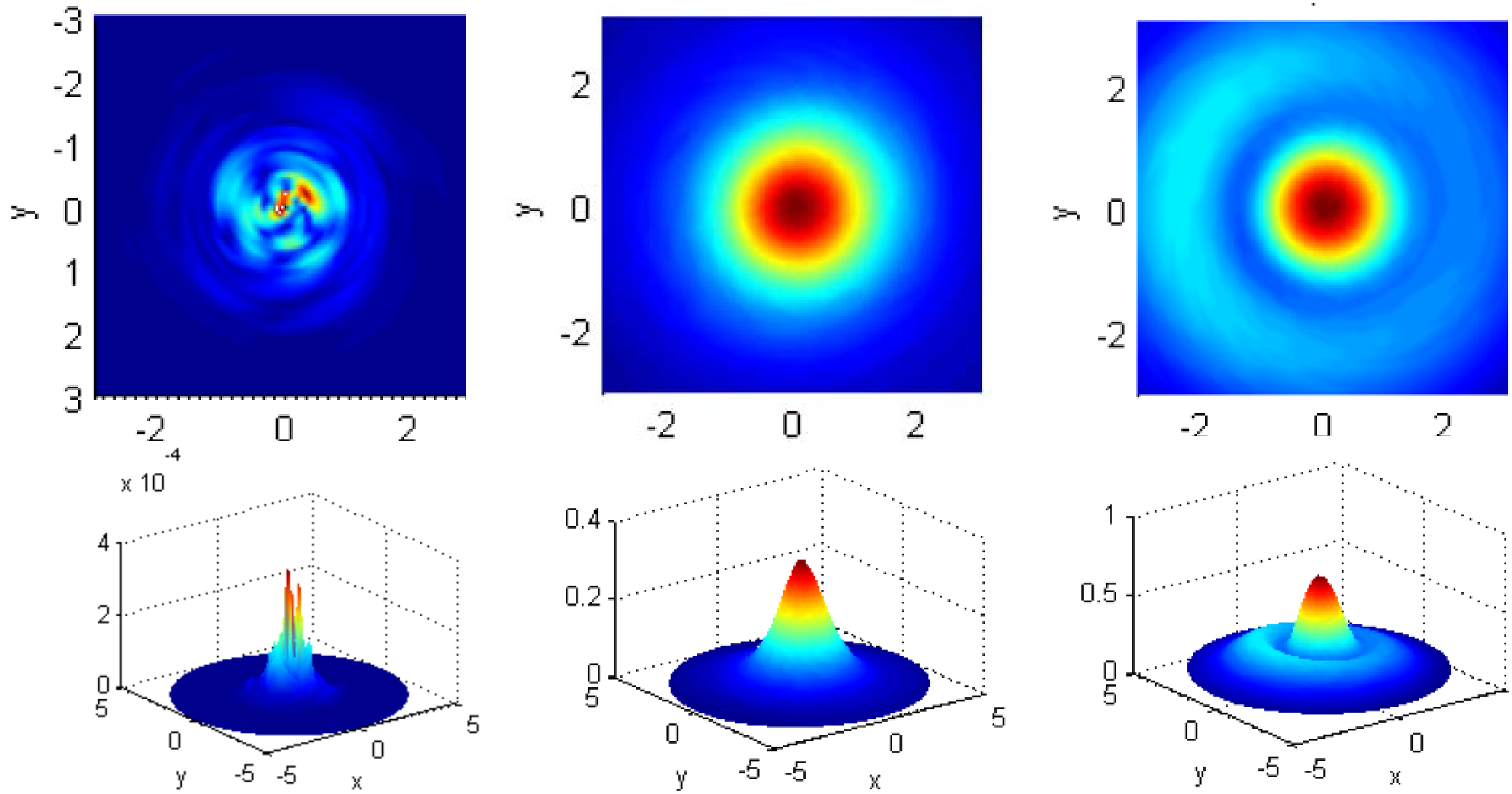
$$\frac{d}{dz}\left(\frac{\delta\gamma}{\gamma}\right)_c = \frac{2r_e\gamma B_w^2}{3m_0c^2}$$

- Increase in energy spread due to spontaneous emission recoil, also known as energy diffusion

$$\frac{d}{dz}\left\langle\left(\frac{\delta\gamma}{\gamma}\right)^2\right\rangle \approx r_e\lambda_C\gamma^2\left(a_w k_w\right)^3$$

Increase in energy spread reduces FEL gain and causes saturation. At $E_b$ > 35 GeV, energy diffusion makes SASE FEL inoperable.

# Transverse Coherence



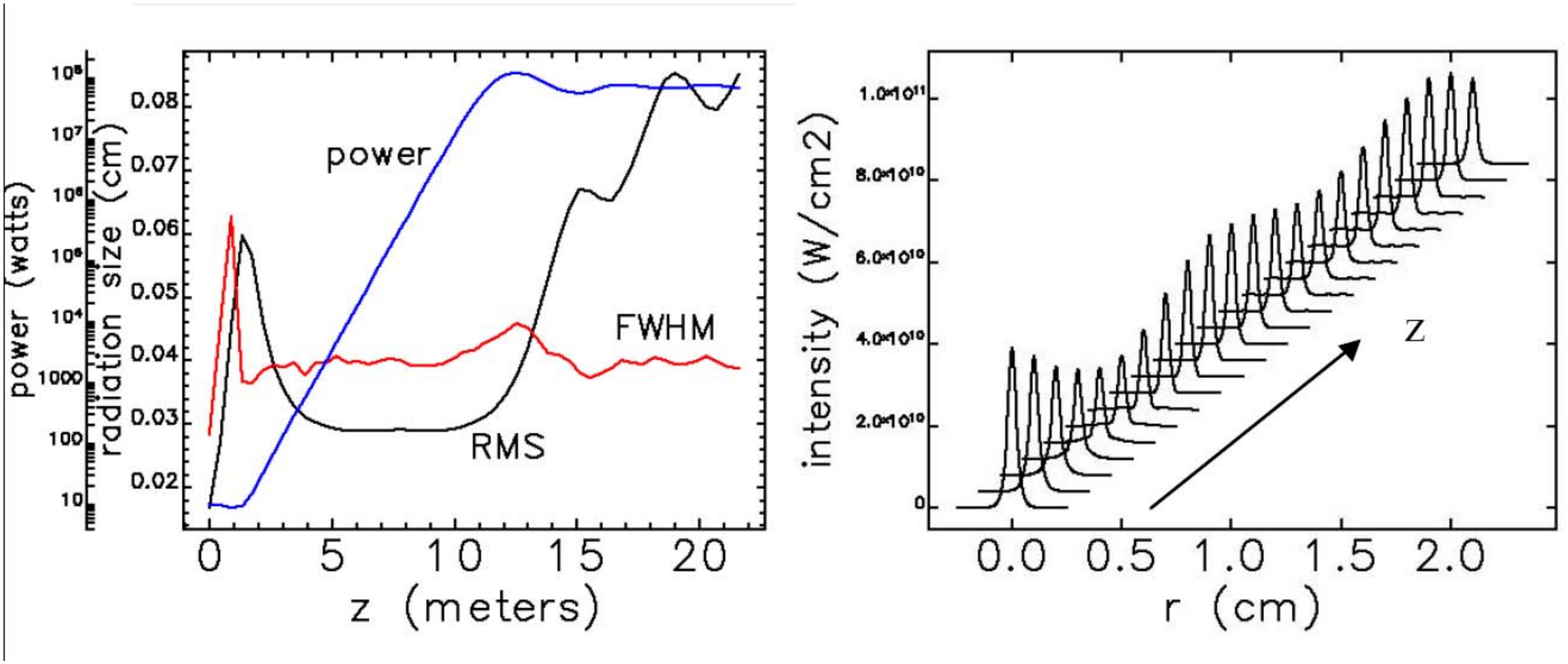

Incoherent Spontaneous Emission

SASE in exponential region

SASE at saturation

Courtesy of I. Zagorodnov

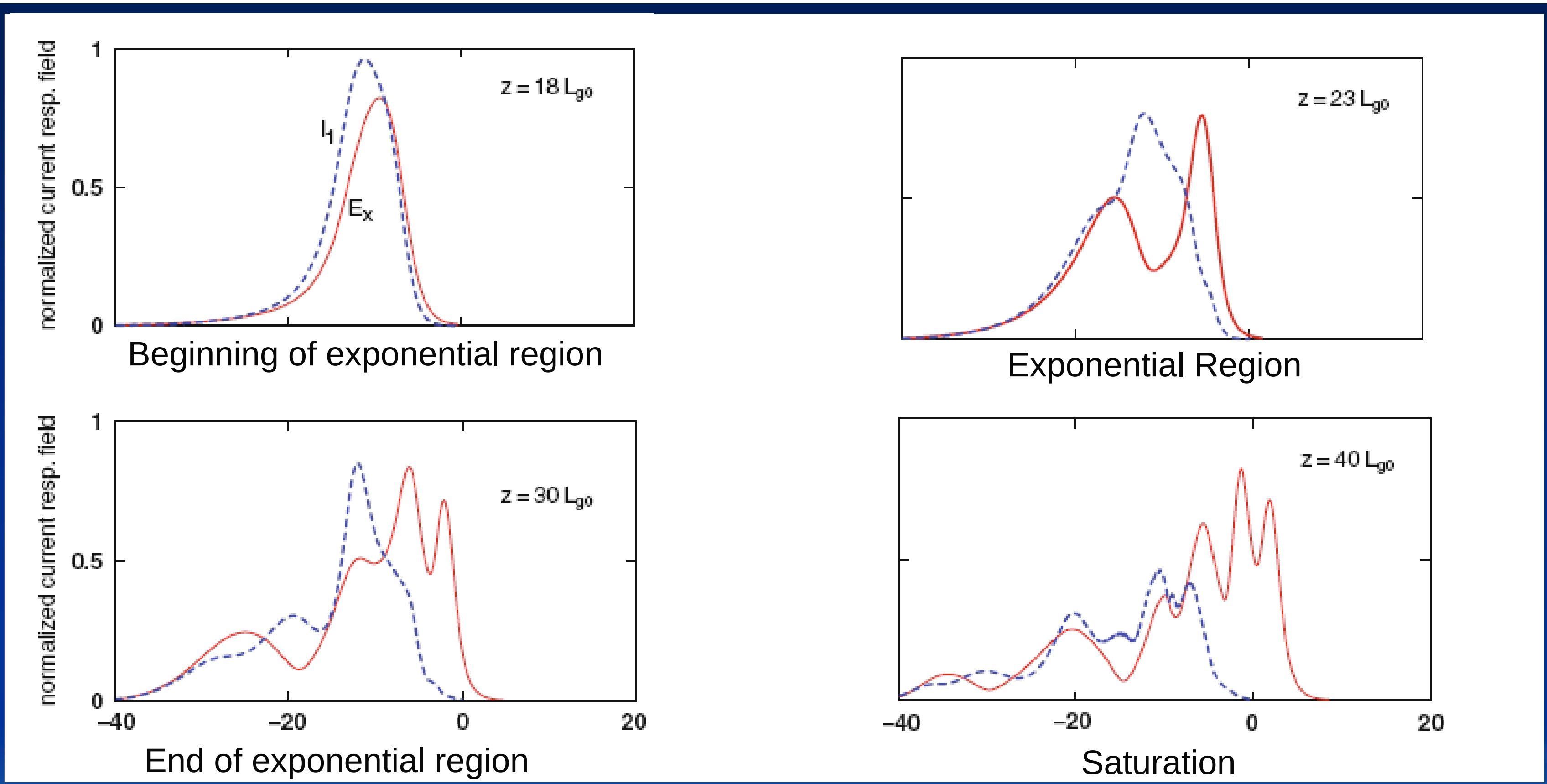
# Saturation reduces optical guiding




Courtesy of Zhirong Huang

The guided mode has constant FWHM. However, the rms radius increases after saturation because more power is added to the wings after the center saturates. Saturation increases the higher order mode content of the SASE optical beam.

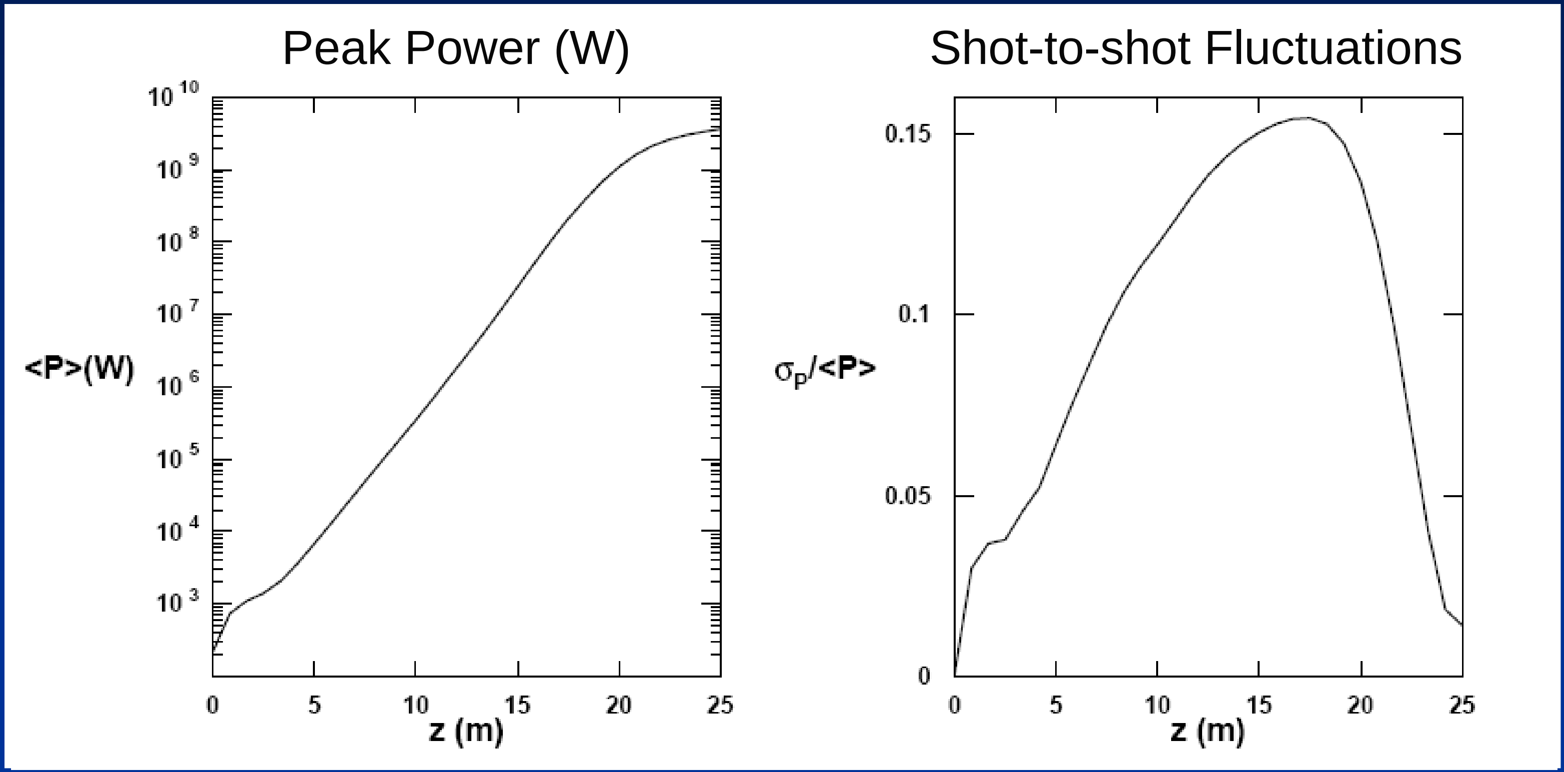
# Limit-Cycle Oscillations




Evolution of electron (blue dash) and FEL (red solid) profiles at FLASH along the wiggler length. The initial profiles are smooth but as the optical pulse slips ahead of the electron bunch, the temporal profile breaks up into sub-pulses.

Courtesy of M. Dohlus

# Power Growth and Fluctuations




Shot-to-shot fluctuations

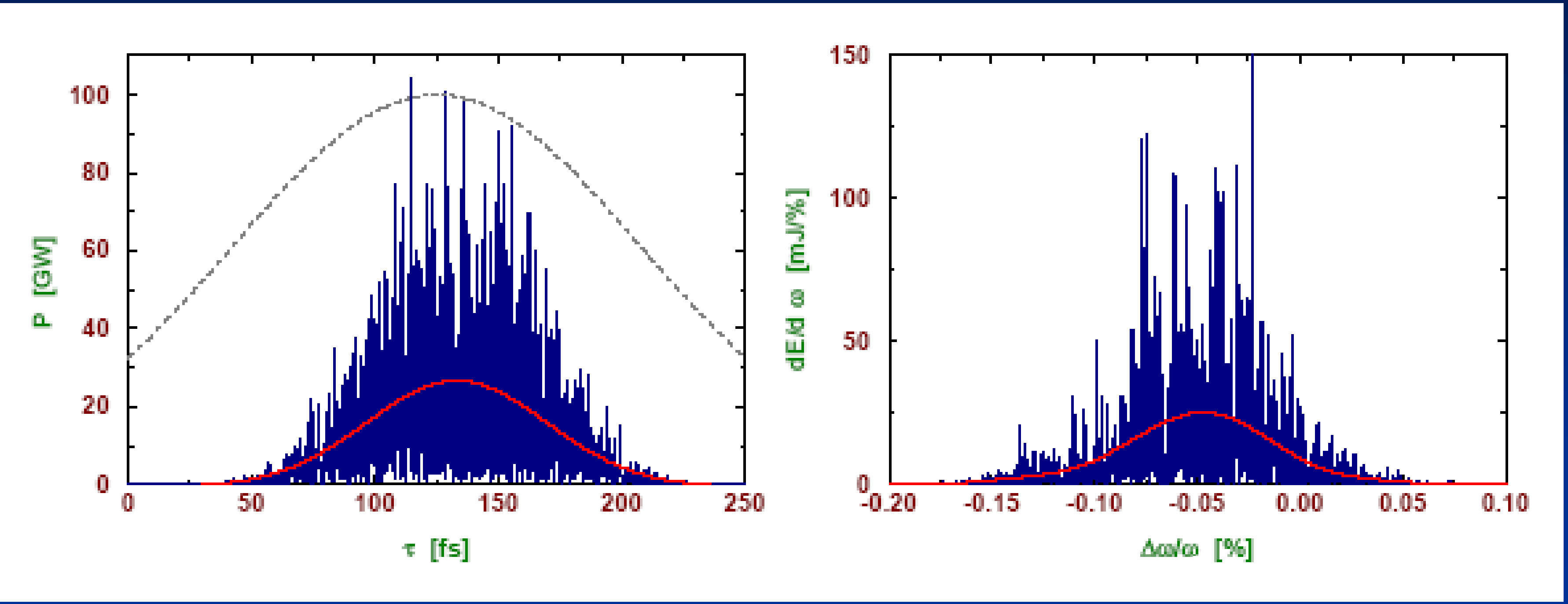
$$\boxed{\frac{\Delta I}{I} = \frac{1}{\sqrt{M}}}$$

$$M = \frac{\sigma_b}{l_c}$$

Bunch length

Coherence length

# Temporal & Spectral Profiles

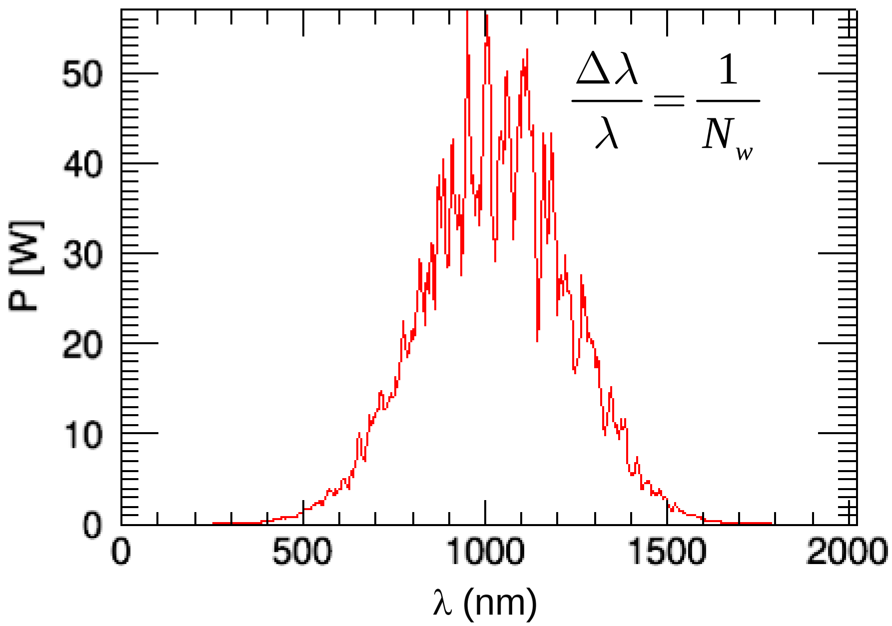



Temporal profile

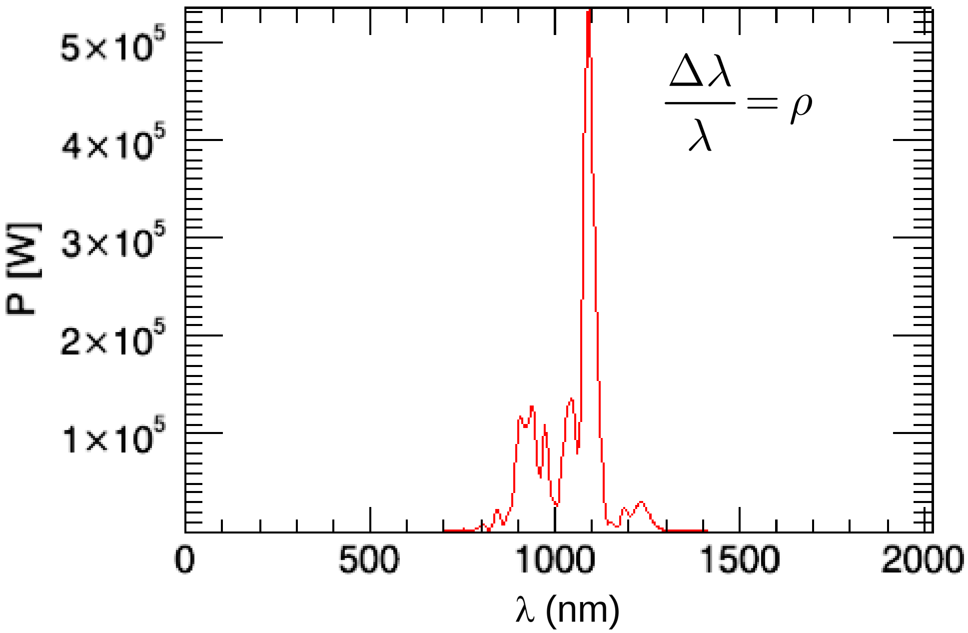
Spectral profile

SASE temporal profile consists of several correlation lengths. The corresponding Fourier transform consists of many spectral features. The spectral FWHM is inversely proportional to the spikes' temporal width while the narrow spectral features are the inverse of the full temporal pulse.

# Longitudinal Coherence




Incoherent Spontaneous Emission

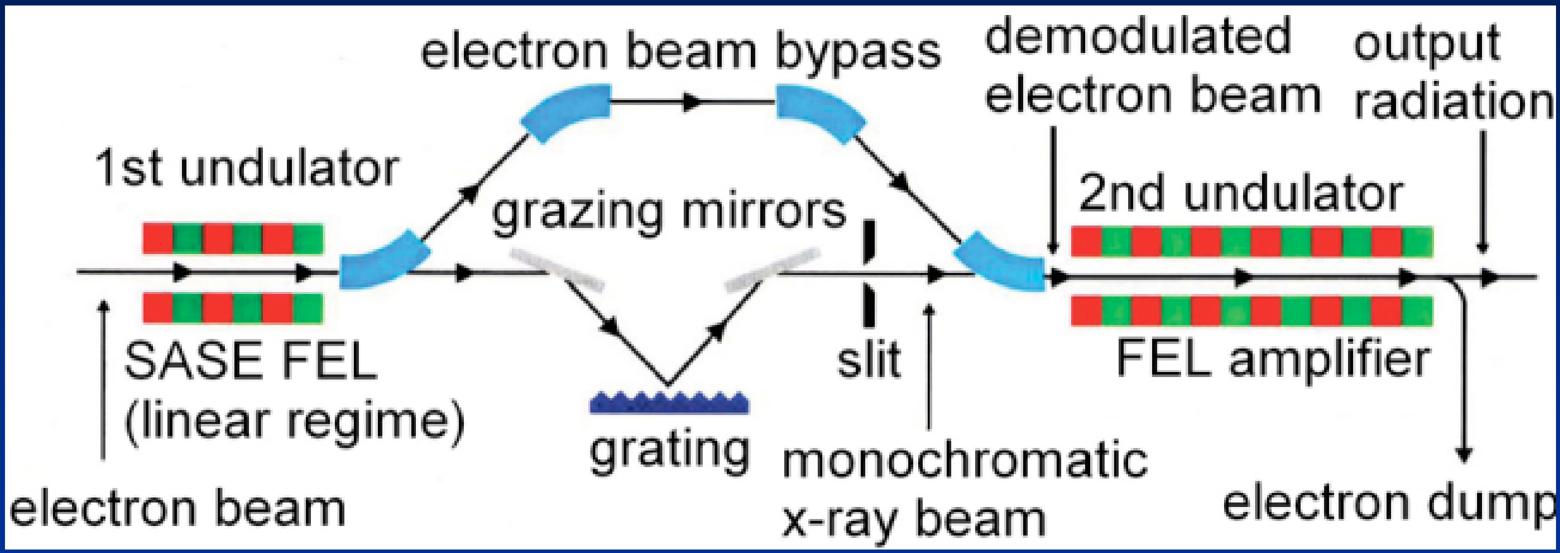



SASE near Saturation

Courtesy of Phil Sprangle

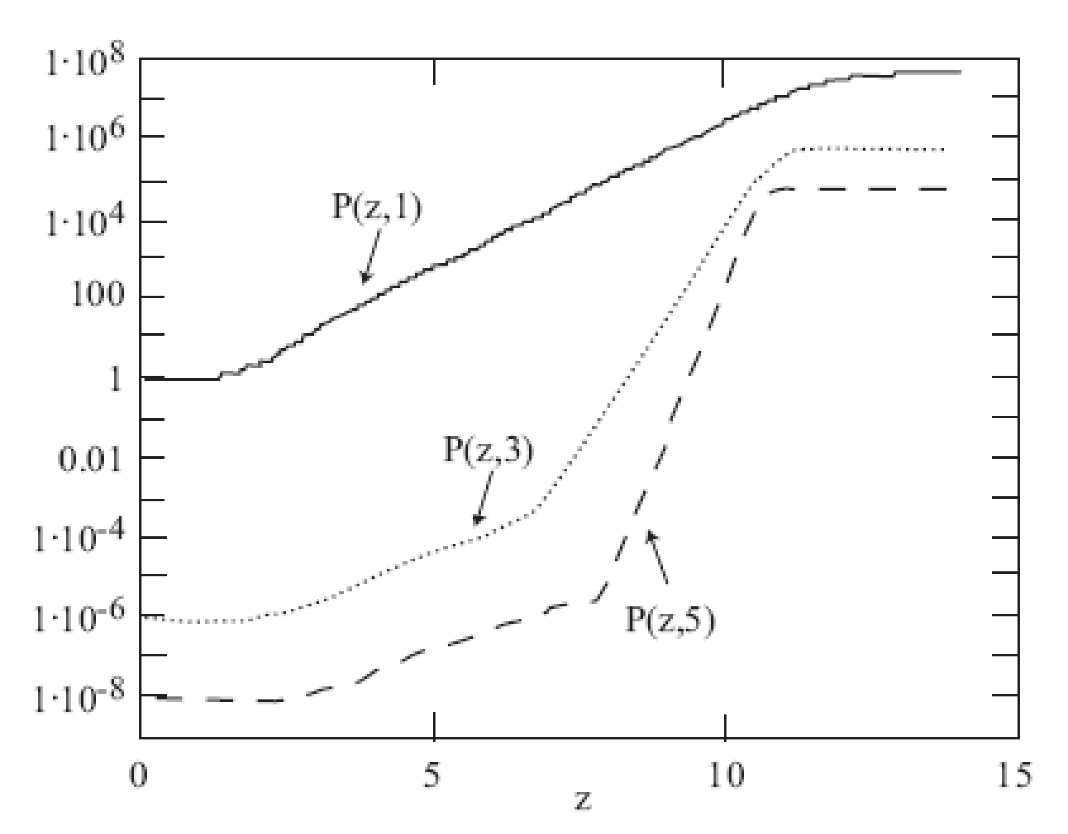
SASE in exponential growth has narrower linewidth than spontaneous noise. At saturation, SASE bandwidth increases to $\rho$, about the same as $1/N_w$. The temporal profile develops "spikes" with short coherence length (large $\Delta\omega/\omega$).

# Improving Longitudinal Coherence




SASE coherence can be improved by dividing the wiggler into two sections. The first wiggler produces partially coherent FEL light that is made more coherent (monochromatic) with a diffraction grating. The monochromatic light is injected into the second wiggler where it seeds the amplification to generate a fully coherent x-ray beam.

# Harmonic Power Growth in SASE




The fundamental optical power causes electrons to bunch. As electrons develop microbunching, the 3rd and 5th harmonics experience higher gain, their power grows more quickly and saturates earlier than the fundamental.

# Seeded Amplifier

Unlike SASE which starts from spontaneous noise, seeded amplifiers use either a **coherent optical input** (seed laser) or an information-encoded electron beam (pre-bunched beam radiation) to start the amplification process. The seed laser power is initially reduced by a factor of 9. The seed power only has to exceed the spontaneous noise emitted into the coherent bandwidth and angle. Seeding improves the FEL longitudinal coherence and reduce the shot-to-shot energy fluctuations. Slippage in a long wiggler must be considered, as it can reduce the optical pulse energy. In a high-gain FEL, the FEL pulse travels slower than *c* and slippage effects are reduced compared to the low-gain cases.

# Seeded Amplifier Power Growth

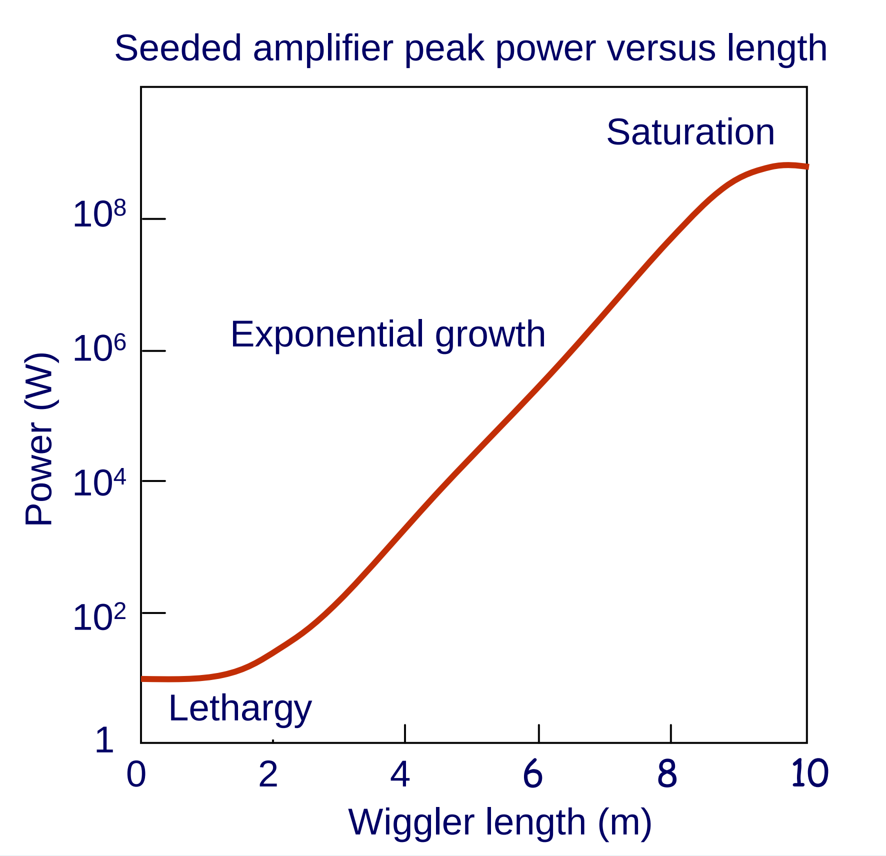
Power grows exponentially with z

$$P(z) = \frac{1}{9} P_0 \exp\left(\frac{z}{L_G}\right)$$

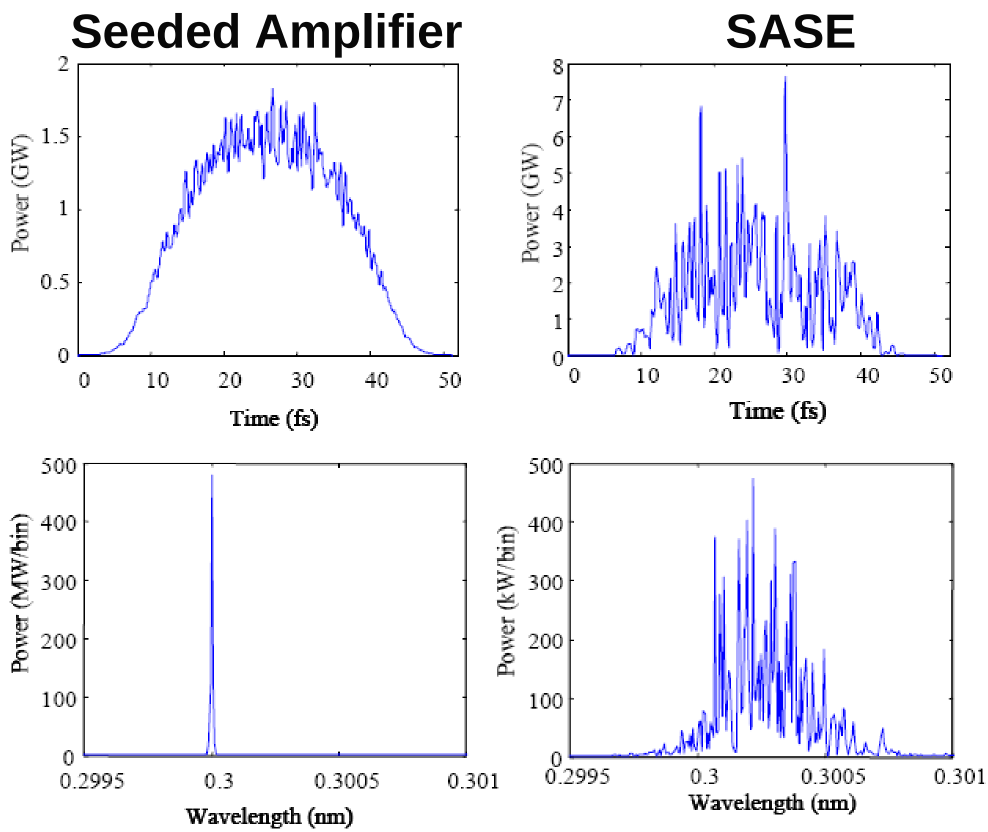
The factor of 1/9 arises because only 1/3 of optical field contributes to the exponentially growing mode.

Seed power must be 9 times the spontaneous power emitted into coherent angle and bandwidth.

$$P_{noise} = \frac{1}{137}\left(\frac{a_w}{1+a_w^2}\right)^2 \frac{h\nu I_b}{e}$$



# Seeded amplifiers have higher longitudinal coherence than SASE




**Temporal Profile**

**Spectrum**

Courtesy of Henry Freund

Longitudinal coherence and fluctuations can be improved by injecting a coherent (long pulse, narrow linewidth) seed to start up the amplification.

# Slippage in a Seeded Amplifier


Slippage in a seeded amplifier varies depending on gain. In the small-signal high gain regime, slippage is reduced due to "**lethargy**" effect. At saturation, lethargy vanishes and the optical pulse slips over the electron bunch one wavelength every wiggler period.

The amplified FEL pulse can undergo limit-cycle oscillations if electron bunch length is much less than the **slippage length** ($N\lambda$). To avoid limit-cycle oscillations that could reduce optical pulse energy, both the electron bunch and optical pulse length must be longer than the slippage length.

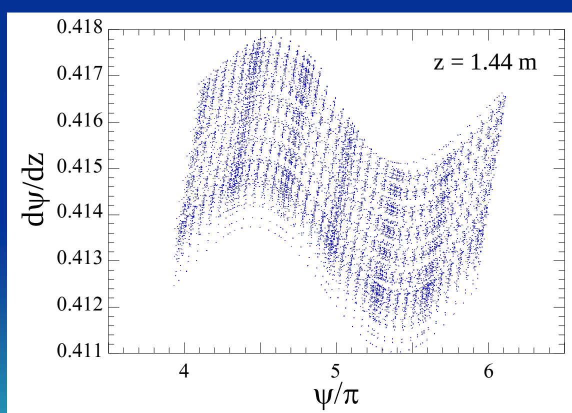
# Optical Klystron and HGHG

Optical klystron FEL uses the same process of energy modulation followed by density modulation in a klystron except the bunching happens at optical wavelengths instead of microwave. Optical klystrons are used mainly to shorten the wiggler length for the same gain (or increase gain for the same length). The optical klystron consists of two wiggler sections separated by a dispersive section. The electrons interact with a seed laser in the first wiggler, undergo synchrotron motion in longitudinal phase space and develop periodic **density modulation** in energy-phase. The modulation period is the seed laser wavelength. This energy-phase modulation transform into density-phase modulation in the dispersive section, which speeds up the high energy electrons and slows down the low energy ones.

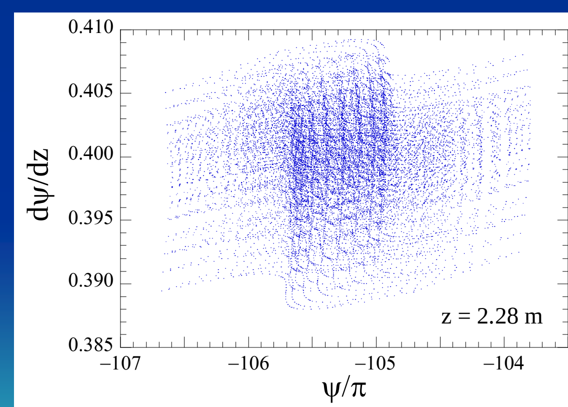
The microbunched electron beams are made to radiate in a second wiggler, either at the fundamental wavelength (conventional OK FEL) or at the harmonics, a process known as **High-Gain Harmonic Generation** (HGHG).

# Principle of an Optical Klystron

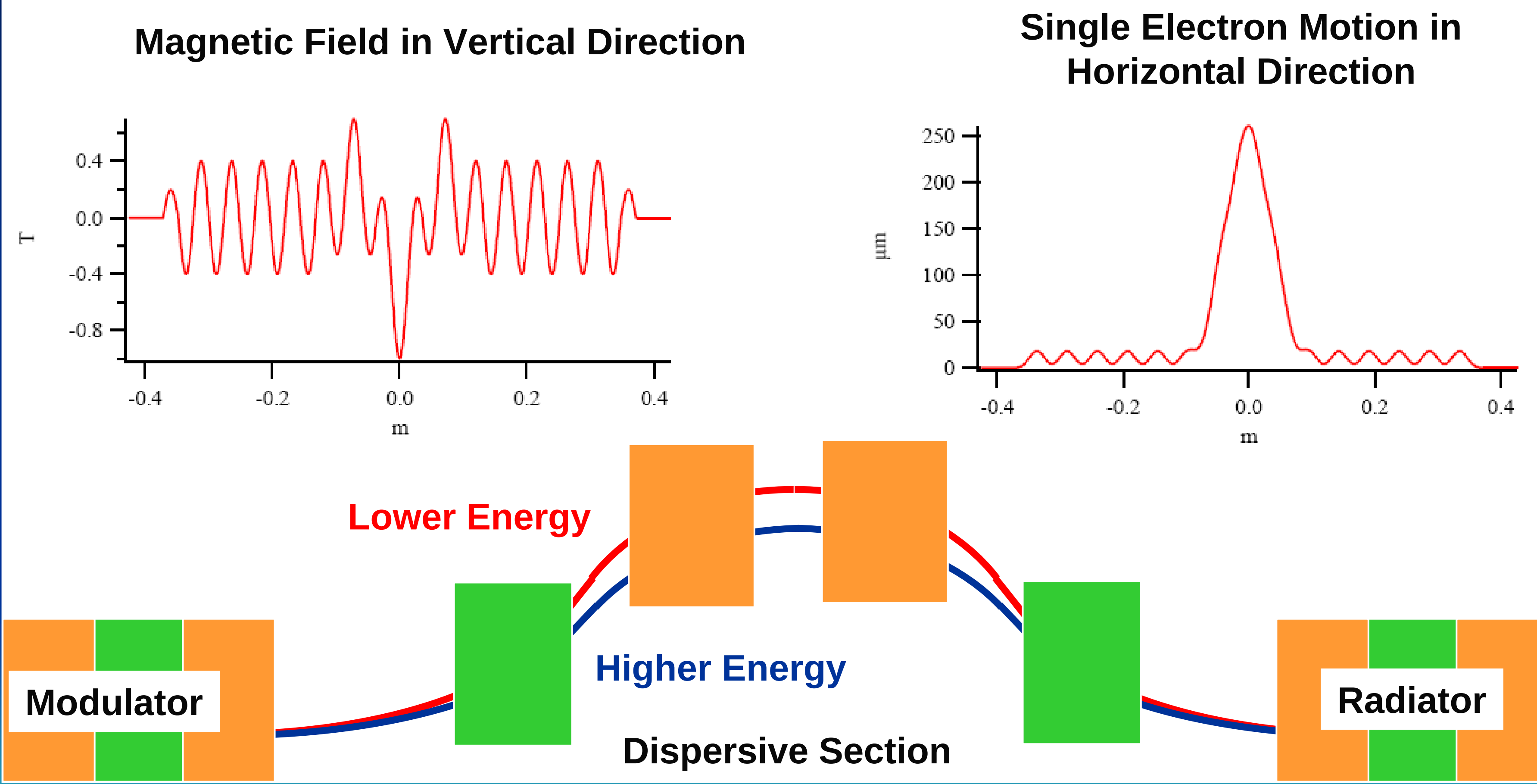

Energy modulation



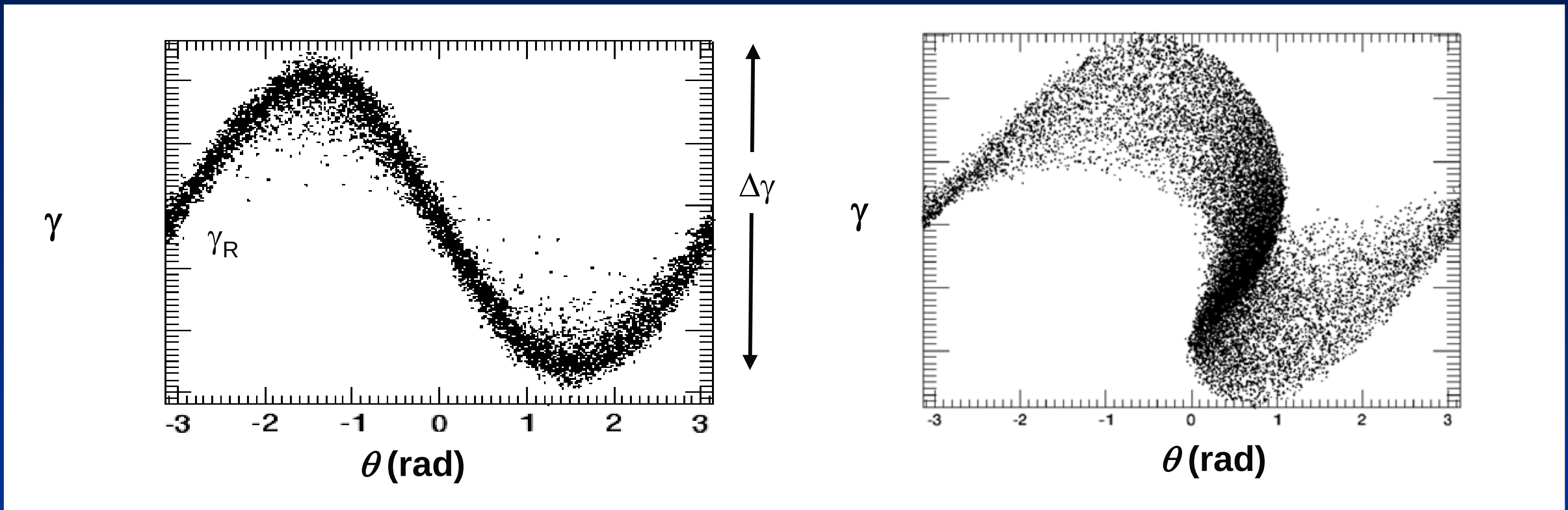

Density modulation



# Magnetic Field and Electron Trajectory in an Optical Klystron



# Energy Modulation & Bunching




Energy modulation at optical field $a_s$

$$\boxed{\frac{\Delta\gamma}{\gamma_R} = 2\sqrt{\frac{a_s a_w}{1+a_w^2}}}$$

Dimensionless field $a_s$

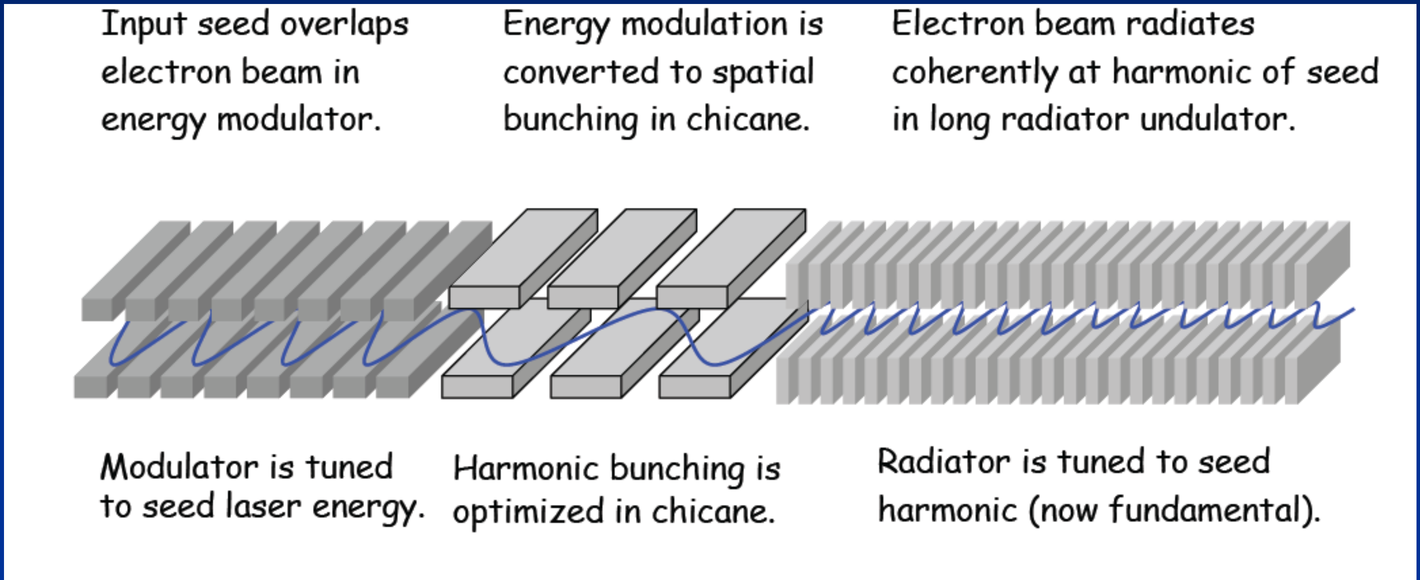
$$a_s = \frac{eE_L}{kmc^2}$$

Laser electric field

$$E_L = \sqrt{2Z_0 I_L}$$

# High-gain Harmonic Generation

HGHG is a way to generate short wavelength radiation using a low energy electron beam and a long-wavelength seed. HGHG requires high quality electron beams (ones with low emittance and energy spread).

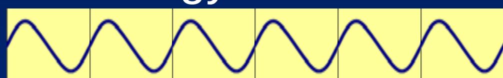



$$\lambda_s = \frac{\lambda_{w1}}{2\gamma^2}\left(1+a_1^2\right)$$

$$\lambda_{FEL} = \frac{\lambda_{wh}}{2\gamma^2}\left(1+a_h^2\right)$$

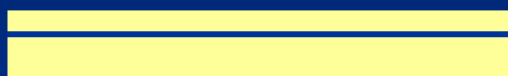
# Harmonic Microbunching

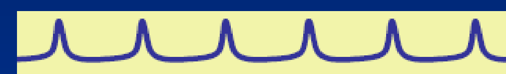
Energy modulation



Density modulation

Energy vs. phase





Density vs. phase

Bunching coefficient at the n^th harmonic

$$\boxed{b_n = J_n\left(nkR_{56}\frac{\Delta\gamma}{\gamma}\right)e^{-\frac{1}{2}\left(n\frac{\sigma_\gamma}{\gamma}\right)^2}}$$

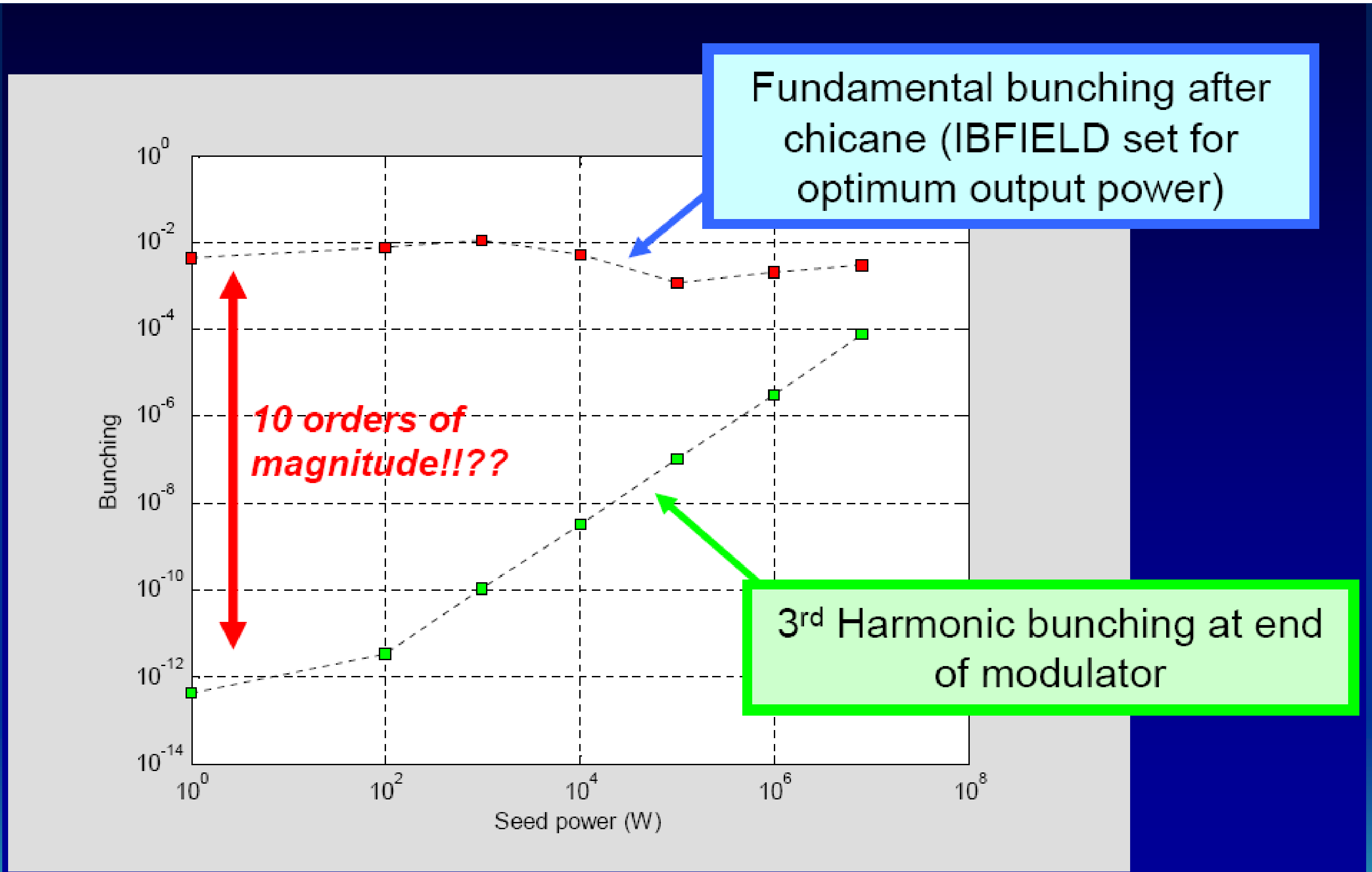
Bessel function of n^th order

Property of dispersive section

Energy modulation

Gaussian spread in energy

# Harmonic Bunching vs Seed Power




Courtesy of N. Thompson
Simulating HGHG with Genesis

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URL


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  http://www.springerlink.com/content/3016v01520372111/fulltext.pdf

- “Simulating HGHG with Genesis 1.3 – An Unofficial Guide” N. Thomson
  http://www.eurofel.org/sites/site_eurofel/content/e160/e167/e190/e193/infoboxContent200/N.Thompson-SimulatingHGHGwithGenesis.pdf